Individual Round — Arithmetic

(1) 191 empty glass bottles must be placed into cases that hold 24 bottles each to be sent to the recycling center. How many cases are required?

(2) A cashier must give a customer 31 cents in change. Using pennies, nickels, dimes and quarters, how many different ways can the change be made?

(3) The human body normally has a 70% water content. A person weighing 200 pounds is stranded in the Great Sandy desert. The scorching heat quickly dehydrates their body. If their body’s water content is reduced to 60% they will die. What would be their weight at that time?
Individual Round — Algebra

(1) If \( x \) is the probability that a given event happens, \( 1 - x \) is called the complementary probability. There are two values of \( x \) such that the complementary probability is equal to twice the square of \( x \). Find both.

(2) A chemist wants to make 200 ml. of 40% sodium pentathol. She has adequate supplies of 10% and of 50% solutions in her lab. How much of each should she mix to create the desired solution?

(3) Given two integers \( a \) and \( b \) we define two other integers in terms of them: \( x = a^2 - b^2 \) and \( y = 2ab \). If \( x \) and \( y \) are the lengths of the sides of a right triangle, find the simplest possible expression for the hypotenuse \( \sqrt{x^2 + y^2} \).
(1) Figure ABCDE is a regular pentagon. Point F is located so that ABF is an equilateral triangle. Find the measure of angle $\angle CBF$.

(2) The right triangle ABC has hypotenuse $\sqrt{18}$, point B lies at the center of a circle and points A and C lie on the circle. Find the area of triangle ABC.

(3) Given that figure $ABCD$ is a square, that $AE = AF = DH = 5$ and that $BF = BG = 12$, find the area of figure $EFGH$. 
You need to construct a set of shelves for outdoor use. The plans (which indicate how the boards are screwed together) are attached. The shelves consist of 2 uprights which are 4 feet long and 3 horizontal members that are 5 feet long. These 5 pieces of wood must be whole pieces – you cannot splice cut boards together. The local lumber yard gives you the following prices for various lengths of 2” by 12” pressure-treated lumber:

<table>
<thead>
<tr>
<th>length</th>
<th>10 feet</th>
<th>12 feet</th>
<th>16 feet</th>
</tr>
</thead>
<tbody>
<tr>
<td>price</td>
<td>$13.29</td>
<td>$16.97</td>
<td>$22.97</td>
</tr>
</tbody>
</table>

(1) How can the shelves be built for minimum cost?

(2) Suppose your neighbor would like the same set of shelves. How can the two sets of shelves be built for the lowest cost?
All boards are 2” by 12” pressure-treated lumber. Assembly requires twelve #14 by 3 inch galvanized wood screws.
Team Round — General

(1) An isosceles triangle has two sides of length 5 and one side of length 6. Find another isosceles triangle with two sides of length 5 (which is not similar to the first) that has the same area.

(2) The digits of the year 2003 add up to 5. How many other 4 digit numbers have digits that sum to 5? (Note that a 4 digit number cannot have a 0 in the thousands place.)

(3) It takes a train 7 seconds to pass by a stationary observer, and 26 seconds to completely traverse the length of a station that is 380 meters long. Find the speed of the train and its length.