

New Haven High School Math Contest — 2007
Individual Round — Arithmetic

(1) What is 75% of $2/3$?

(2) The product of two distinct positive integers is 42. What is the smallest possible sum of these integers?

(3) What is the value of the following alternating sum?

$$1 - 2 + 3 - 4 + 5 - 6 + \dots + 2005 - 2006 + 2007$$

(4) How many four digit whole numbers have an odd digit in the thousands place, an even digit in the hundreds place, and have all four digits different?

New Haven High School Math Contest — 2007
Individual Round — Algebra

(1) Find a value for x that satisfies the equation $3x + 2 = 10 - x$.

(2) Find both values of x that satisfy $|1 - x| = 6$.

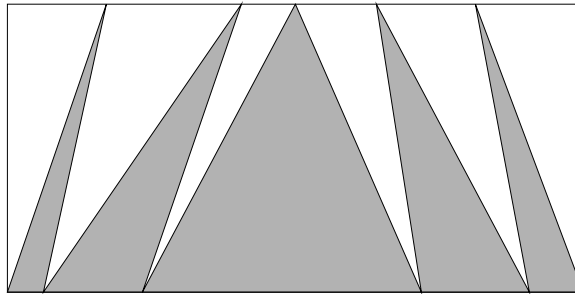
(3) For what values of x does the reciprocal of $x + 1$ equal $x - 1$?

(4) If $f(x) = x^2 - 3x - 5$, what are the values of k such that $f(k) = k$?

New Haven High School Math Contest — 2007
Individual Round — Geometry

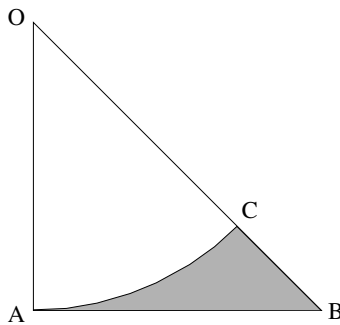
- (1) Recall that a triangle whose sides are 3, 4 and 5 inches long is a right triangle. What is its area?

- (2) What fraction of the area of the rectangle illustrated below is shaded?



- (3) A wooden cube has an edge length of 4 inches. The cube is painted red all over and then cut up into 64 little 1 by 1 by 1 cubes. How many of these little cubes have exactly one face painted red?

- (4) In the figure below, AC is an arc of a circle centered at O . The line segments \overline{OA} and \overline{AB} are perpendicular and both have length 12 centimeters. What is the area of the shaded region?



New Haven High School Math Contest — 2007
Team Round — CAPT

JoAnne has landed a great job which begins right after graduation. She will be commuting 15,000 miles each year, so she needs to buy a fuel efficient car for commuting. She is considering three different models: the Toyota Prius Hybrid, the Honda Civic Hybrid, and the Honda Civic DX (a traditional 4 cylinder engine). For the purpose of making comparisons JoAnne assumes that she will keep her car for five years, so she needs to compute the total cost of purchasing and operating each vehicle over a five year period.

The purchase prices and estimated gas mileages of these three vehicles are given in the following table.

Make/Model	Price	Mileage
Toyota Prius Hybrid	\$22,175	55 mpg
Honda Civic Hybrid	\$22,600	50 mpg
Honda Civic DX	\$18,710	35 mpg

- (1) Assuming gasoline costs \$3.00 per gallon, which of the two hybrid models will be cheaper to purchase and operate over a 5 year period?
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- (2) JoAnne decides she doesn't like the appearance of the Honda Civic Hybrid, so she just needs to decide between the Prius and the Civic DX. If gas is cheap enough, she knows that the lower purchase price of the Civic DX should make it cost less to purchase and operate over a five year period than the Prius. On the other hand, if gasoline becomes very expensive the Prius will be cheaper. At what price per gallon for gasoline does the Prius become cheaper than the Civic DX? (We will refer to this as the "break even price.")
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- (3) If we change our assumptions so that JoAnne needs to drive 25,000 miles per year, what is the break-even gasoline price (between the Civic DX and the Prius)?
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- (4) Express the break-even gasoline price (between the Civic DX and the Prius) as a function of the number of miles driven per year.
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- (5) JoAnne realizes she will need to drive some number of miles each year in addition to her commute. Assume that gas stays at \$3.00 per gallon. What would be the total number of miles (over the entire five year period) that would give the same total costs for the Prius and the Civic DX?
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New Haven High School Math Contest — 2007
Team Round — General

- (1) What is the sum of all the odd numbers from 1 to 2007?

$$1 + 3 + 5 + \dots + 2005 + 2007 =$$

- (2) How many integers between 0 and 1,000 are multiples of 4 and **do not** contain any of the digits 6, 7, 8, 9 or 0?
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- (3) Suppose a function $f(x)$ has the property that $f(1/x) - 2 \cdot f(x) = x$ for all values of x , other than 0. What is $f(2)$?
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- (4) A *Pythagorean triple* is an ordered triple of positive integers which form the sides of a right triangle. (For example, (3, 4, 5) is a Pythagorean triple.) Determine a Pythagorean triple such that one of the integers is 2007.
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