Name ______________________ Section ____________ Date ____________

Laboratory 13

The Ballistic Pendulum and Projectile Motion

PRELABORATORY ASSIGNMENT

Read carefully the entire description of the laboratory and answer the following questions based on the material contained in the reading assignment. Turn in the completed prelaboratory assignment at the beginning of the laboratory period prior to the performance of the laboratory.

1. What are the conditions under which the total momentum of a system of particles is conserved?

2. What kind of collision conserves kinetic energy?

3. What kind of collision does not conserve kinetic energy? What kind of collision results in the maximum loss of kinetic energy?

4. A ball of mass 0.0750 kg is fired horizontally into a ballistic pendulum as shown in Figure 13.1. The pendulum mass is 0.3500 kg. The ball is caught in the pendulum, and the center of mass of the system rises a vertical distance of 0.145 m in the earth's gravitational field. What was the original speed of the ball? Assume $g = 9.80\ \text{m/s}^2$.

5. How much kinetic energy was lost in the collision of problem 4?

6. A projectile is fired in the earth's gravitational field with a horizontal velocity of $v = 9.00$ m/s. How far does it go in the horizontal direction in 0.550 s?

7. How far does the projectile of question 6 fall in the vertical direction in 0.550 s?

8. A projectile is launched in the horizontal direction. It travels 2.050 m horizontally while it falls 0.450 m vertically, and it then strikes the floor. How long is the projectile in the air?

9. What was the original velocity of the projectile described in question 8?

10. What is the velocity in the horizontal direction of the projectile in question 8 when it strikes the floor? What is its velocity in the vertical direction at this time? What is the magnitude of its velocity as it strikes the floor?

Laboratory 13

The Ballistic Pendulum and Projectile Motion

OBJECTIVES

In collisions, the forces involved are internal, and therefore momentum is conserved. A collision is called "elastic" if kinetic energy is also conserved. An *inelastic collision* is one in which some kinetic energy is lost. If the colliding particles stick together, the collision is called "completely inelastic," and the maximum possible loss of kinetic energy occurs. A ballistic pendulum is a device used to measure the velocity of a projectile fired into an initially stationary pendulum. The pendulum is designed to catch the ball, causing a completely inelastic collision. Measurements for the ball fired into the pendulum and for the ball fired horizontally as a free projectile will be used to accomplish the following objectives:

1. Determination of the initial velocity of the ball and the initial velocity with which the pendulum plus ball moves after the collision
2. Determination of the kinetic energy loss in the collision of the ball with the pendulum
3. Independent determination of the initial velocity of the ball by firing it as a free projectile
4. Comparison of the velocity of the ball determined by the two different experimental arrangements

EQUIPMENT LIST

1. Ballistic pendulum apparatus with projectile ball
2. Laboratory balance and calibrated masses
3. Meter stick, plain paper, carbon paper, and masking tape

THEORY

According to the principle of conservation of momentum, if there are no external forces acting on a system of several particles, then the total momentum of the system remains constant. Collision processes are particularly good examples of this concept. In this laboratory, a ballistic pendulum will be used to measure the velocity of a ball projected by a spring gun. Consider the process shown in Figure 13.1 in which a ball of mass m moving initially in the horizontal direction with speed v_{x0} strikes a pendulum designed to catch the ball. The pendulum of mass M, upon receiving the ball, swings about a pivot point O to some maximum height y_2 that is greater than its original height y_1. The system of ball plus pendulum rises a vertical distance of $y_2 - y_1$ as a result of the process.

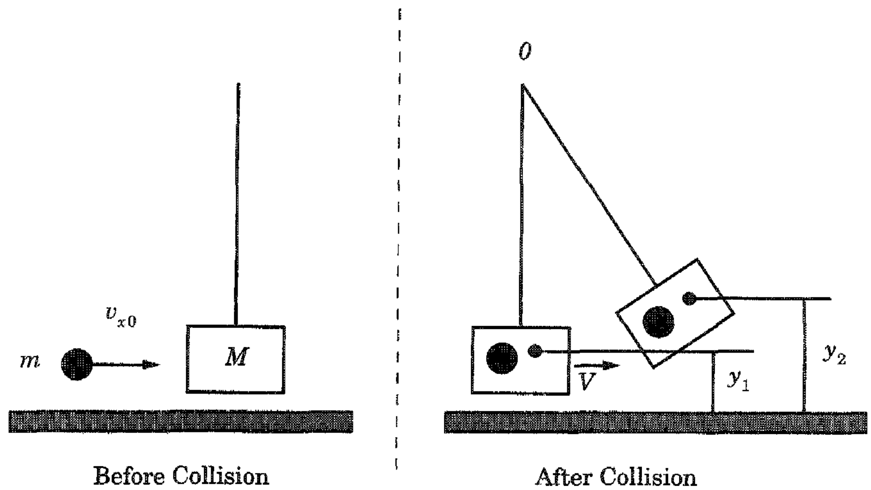

Figure 13.1 Ballistic pendulum of mass *M* before and after collision with ball of mass *m*.

The analysis of the event is best done in two steps. Momentum is conserved in the collision because the only forces acting on the ball and the pendulum in the direction of motion are the forces of the collision. The collision is completely inelastic because the two particles stick together after the collision and thus move with the same velocity V. The equation for conservation of momentum is

$$mv_{x0} = (m + M)V \tag{1}$$

The collision itself does not conserve mechanical energy. In fact, the original kinetic energy $[\frac{1}{2}\,mv_{x0}^2]$ is much greater than the kinetic energy immediately after the collision $[\frac{1}{2}\,(m + M)V^2]$. On the other hand, mechanical energy is conserved in the motion of the combined mass (ball plus pendulum). The combined mass moves with velocity V immediately after the collision. It swings up along the arc of the pendulum. It rises a vertical distance of $y_2 - y_1$ and comes to rest instantaneously at that maximum height as the kinetic energy immediately after the collision is converted into gravitational potential energy. In equation form, $\frac{1}{2}(m + M)V^2 = (m + M)g(y_2 - y_1)$. Solving for V gives

$$V = \sqrt{2g(y_2 - y_1)} \tag{2}$$

If equation 1 is solved for the initial velocity of the ball, the result is

$$v_{x0} = \left(\frac{m + M}{m}\right)V \tag{3}$$

Therefore, a measurement of the height $y_2 - y_1$ leads to a determination of V using equation 2, and with V known, equation 3 can be used to determine v_{x0}.

EXPERIMENTAL PROCEDURE—BALLISTIC PENDULUM

1. Slide the projectile ball (which has a hole in it) onto the rod of the spring gun (Figure 13.2). The ball may have been stored in the pendulum bob. If so, when removing the ball be careful to push up on the spring in the bob that serves as a mechanism to catch the ball. This spring can be easily broken if the ball is forced out

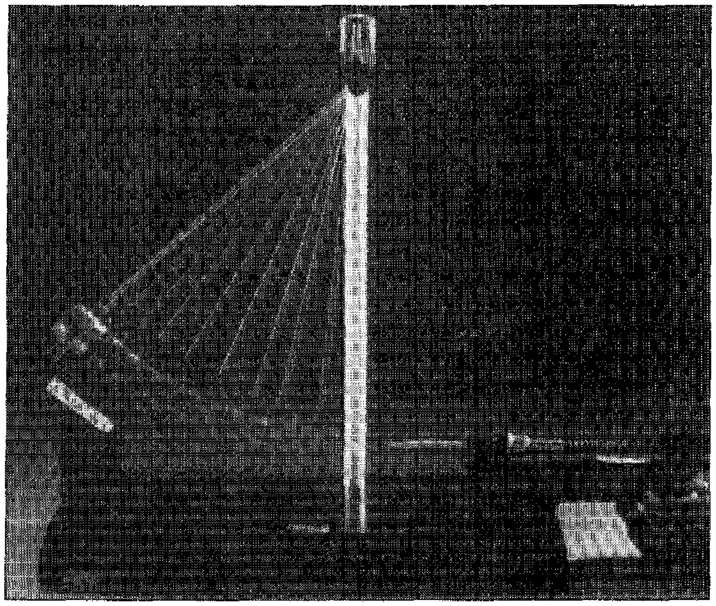

Figure 13.2 Ballistic pendulum apparatus. (Photo courtesy of Central Scientific Co., Inc.)

of the bob without releasing the spring first. When the ball has been placed on the rod, cock the gun by pushing against the ball until the latch catches. *Be very careful not to get your hand caught in the spring gun mechanism.* Fire the gun several times to see how it operates. There are two common problems. If the ball does not catch in the pendulum bob, the spring in the bob should be adjusted or replaced. If the pawl that is designed to catch on the notched track does not engage, the pendulum suspension should be adjusted by means of the screws at the suspension points. The alignment of this suspension is critical and frequently requires adjustment.

2. A sharp curved point on the side of the pendulum (or on some models a dot) marks the center of mass of the pendulum-ball system. Let the pendulum bob hang vertically and measure the distance y_1 of the point marking the center of mass above the base of the gun (Figure 13.1). Record the value of y_1 in Data Table 1.

3. Place the ball on the rod, push against the ball to cock the gun, and fire the ball into the stationary pendulum while it hangs freely at rest. The pendulum will catch the ball, swing up, and then lodge in the notched track. Record in Data Table 1 the position number p at which the pawl on the pendulum catches on the track. Measure the distance y_2 of the center of mass point above the base of the apparatus (Figure 13.1). Record the value of y_2 in Data Table 1. Repeat this procedure four more times for a total of five trials, recording the position p and measuring the distance y_2 for each trial.

4. Loosen the screws holding the pendulum in its support and remove the pendulum consisting of the rod and the bob. Determine the mass of the pendulum (bob and rod) using a laboratory balance. Record the pendulum mass as M in Data Table 1. Determine the projectile ball mass and record it in Data Table 1 as m.

CALCULATIONS—BALLISTIC PENDULUM

1. Calculate the distance $y_2 - y_1$ that the combined mass rises for each trial. Record these values in Calculations Table 1.

2. Using equation 2, calculate the velocity V of the combined mass for each of the five trials and record them in Calculations Table 1.
3. Using equation 3, calculate the initial speed of the projectile ball v_{x0} for the five trials and record those values in Calculations Table 1.
4. Calculate the mean $\overline{v_{x0}}$ and the standard error α_v for the five values of v_{x0}. Record them in Calculations Table 1.

THEORY—PROJECTILE MOTION

The ballistic pendulum apparatus can be used to launch the ball as a free-falling projectile. If the pendulum is raised and the pawl placed in one of the notches on the track before the gun is fired, the ball will travel a path like that shown in Figure 13.3. The ball will travel a horizontal distance X while it falls a vertical distance Y.

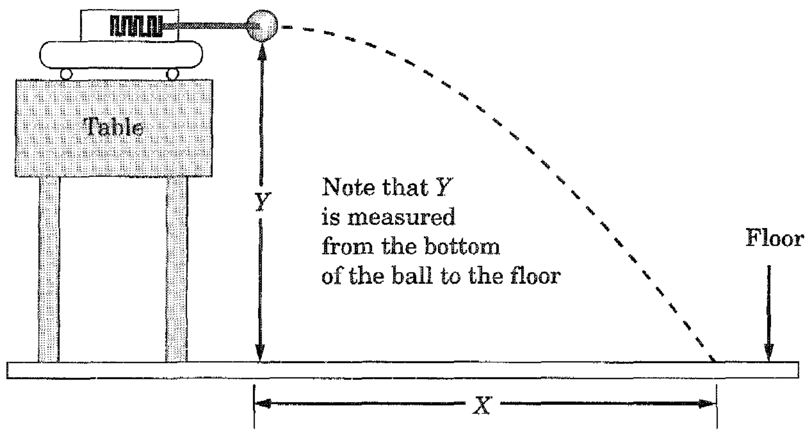

Figure 13.3 Motion of the ball moving horizontal distance X while free-falling height Y.

The original velocity of the ball is completely in the x direction with no y component. The acceleration due to gravity is the only acceleration of the ball, and it is in the y direction with no x component. The equations for the horizontal displacement X and the vertical displacement Y as a function of the time t after the ball is launched are

$$X = v_{x0}\, t \tag{4}$$

$$Y = \tfrac{1}{2} g t^2 \tag{5}$$

Note that equation 5 has been written so that a positive displacement is downward in the same direction as g. Equations 4 and 5 can be combined to eliminate the time t and thus express v_{x0} in terms of X and Y as

$$v_{x0} = \frac{X}{\sqrt{2Y/g}} \tag{6}$$

Equation 6 can be used to determine the initial velocity v_{x0} by firing the projectile from a known height Y and measuring the value of X that results.

The velocity in the y direction is initially zero. As the projectile falls under the influence of gravity, it acquires a velocity in the y direction. The magnitude of the velocity in the y direction is given by

$$v_y = gt = \sqrt{g\, 2Y/g} = \sqrt{2gY} \tag{7}$$

EXPERIMENTAL PROCEDURE—PROJECTILE MOTION

In the following procedure, be extremely careful not to fire the ball when anyone is in a position to be struck by it. Serious injury could result.

1. Raise the pendulum and let it catch on the notched track so that the ball can be fired under it.
2. Place the apparatus near the front edge of the laboratory table in a place where there is room for the ball to strike the floor before hitting a wall or any other object. It is crucial that the gun be fired each time from the same position relative to the table. It may be necessary to clamp the apparatus to the table. Place a piece of heavy cardboard or some other object in a position to catch the ball after it strikes the floor but before it strikes a wall. *Do not allow the ball to strike a wall because it will likely damage it.* Make several test firings to locate the approximate place where the ball will land on the floor.
3. Place a sheet of white paper on the floor approximately centered where test firings have landed. Place a piece of carbon paper over the white paper in such a way that the ball striking the carbon paper will leave a dot on the white paper. Tape both of the papers to the floor.
4. Place the ball on the rod of the spring gun. The vertical distance Y that the ball will fall is the distance from the *bottom* of the ball to the floor as shown in Figure 13.3. Measure this distance to the nearest 0.1 mm and record it in Data Table 2 as Y.
5. Fire the ball five times onto the same sheet of paper. Place the ball on the rod and measure the horizontal distance X to the nearest 0.1 mm from the center of the ball to the center of each dot on the paper. Record these five values of X in Data Table 2.

CALCULATIONS—PROJECTILE MOTION

1. Using equation 6, calculate the value of v_{x0} for each of the five values of X. Use a value of $g = 9.80\ \text{m/s}^2$. Record these values in Calculations Table 2.
2. Calculate the mean $\overline{v_{x0}}$ and standard error α_v for the five values of v_{x0} and record them in Calculations Table 2.

Name ______________________ Section ____________ Date ____________

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LABORATORY REPORT

Data Table 1

Trial	p	y2 (m)
1		
2		
3		
4		
5		

$m =$ kg	$M =$ kg	$y_1 =$ m

Calculations Table 1

$y_2 - y_1$ (m)	V (m/sec)	v_{x0} (m/sec)

$\overline{v_{x0}} =$ m/sec	$\alpha_v =$ m/sec

Data Table 2

Trial	X (m)
1	
2	
3	
4	
5	

$Y =$ m

Calculations Table 2

v_{x0} (m/sec)

$\overline{v_{x0}} =$ m/sec	$\alpha_v =$ m/sec

SAMPLE CALCULATIONS

QUESTIONS

1. Compare the two different values of $\overline{v_{x0}}$. Calculate the difference and the percentage difference between them. Do the two measurements agree within the combined standard errors of the two measurements?

2. Is either of the two measurements of $\overline{v_{x0}}$ more accurate than the other? Is either of these measurements more precise than the other? State clearly the basis for your answer in each case.

3. Calculate the loss in kinetic energy when the ball collides with the pendulum as the difference between the kinetic energy before and immediately after the collision.

4. What is the fractional loss in kinetic energy? Find by dividing the loss calculated in question 3 by the original kinetic energy.

5. Calculate the ratio $M/(m + M)$ for the values of m and M in Data Table 1. Compare this ratio with the ratio calculated in question 4. Theoretically these two ratios should be the same. State the level of agreement for these two quantities for your data.

6. Consider the ball when fired as a free-falling projectile as it hits the ground. What is the velocity of the projectile in the horizontal direction? What is the velocity of the projectile in the vertical direction? What is the magnitude of its velocity as it hits the ground?

7. We have assumed that the ball when fired as a projectile initially moved exactly in the horizontal direction. If in fact the projectile was fired at some small angle θ to the horizontal, the correct equation relating v_0, θ, X, and Y would then be

$$(4.90X^2/v_0^2)\tan^2\theta - X\tan\theta + (Y + 4.90X^2/v_0^2) = 0$$

Assume that the v_0 determined by the ballistic pendulum measurement is correct. Use that value of v_0 and the measured X and Y in the above equation and solve for the angle. If the ball was fired at that angle relative to the horizontal it would be enough to account for the difference in the two measured values of v_0. (Note the equation is a quadratic in $\tan\theta$ and note that Y is negative.)