Centripetal Acceleration of an Object in Circular Motion

OBJECTIVES

- ❑ Investigate how the period T of an object that rotates in a circle is related to the mass of the object M, speed v, and radius R of the circle.
- ❑ Determine the centripetal force F as the force required to stretch a spring.

EQUIPMENT LIST

- Hand-operated centripetal force apparatus (The device described is available from Sargent-Welch Scientific Company.)
- Laboratory balance, calibrated slotted masses, mass holder, laboratory timer, and metal ruler

THEORY

An object moving in a circle at constant speed has a velocity vector that is always tangent to the circle. The direction of the velocity is continuously changing. The object is accelerated because acceleration is by definition a change in velocity per unit time. Figure 16-1 shows the velocity vector at points around the circle for an object moving in a circle at constant speed. The lengths of the vectors are the same because the speed is constant, and the direction of the vectors indicates the direction of the velocity at that point. Also shown in Figure 16-1 are the velocity vectors $\mathbf{v}_i$ and $\mathbf{v}_f$ at two times t_i and t_f. In the third part of the figure is the vector difference $\Delta\mathbf{v} = \mathbf{v}_f - \mathbf{v}_i$ indicating that the change in velocity $\Delta\mathbf{v}$ always points toward the center of the circle. The **acceleration a** is

$$\mathbf{a} = \Delta\mathbf{v}/\Delta t \qquad \text{(Eq. 1)}$$

The acceleration **a** is in the direction of $\Delta\mathbf{v}$. It points toward the center of the circle and has magnitude

$$a = v^2/R \qquad \text{(Eq. 2)}$$

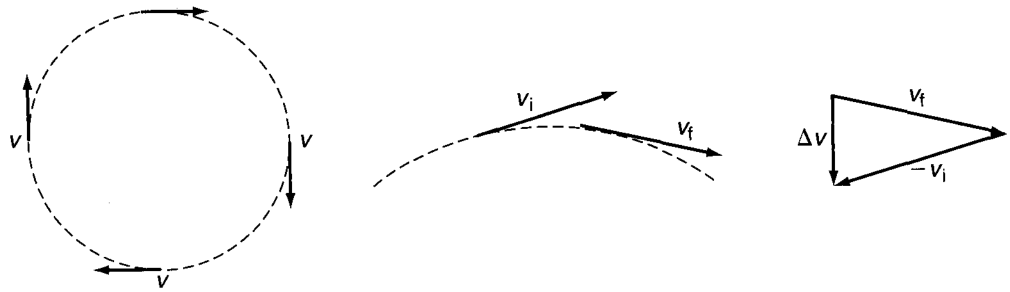

Figure 16-1 Velocity vectors for circular motion at constant speed. Vectors at two times t_i and t close together and the change in velocity $\Delta\mathbf{v}$ pointing toward the center of the circle.

By Newton's second law the centripetal force F and the **centripetal acceleration** a are related by $F = Ma$ where M is the mass of the object moving in a circle at speed v. Using Equation 2 gives

$$F = mv^2/R \qquad \text{(Eq. 3)}$$

The time for one complete revolution around the circle is the period T, which is related to the speed v by the expression

$$v = (2\pi R)/T \qquad \text{(Eq. 4)}$$

The centripetal force apparatus has a mass bob with a pointed tip at the bottom suspended from a horizontal rotating bar. The bob has a spring hooked between the side of the bob and the central rotating shaft. The spring provides a horizontal centripetal force when the bob rotates in a horizontal plane. The bob rotates at a fixed radius R from the central rotating shaft when the tip of the bob passes over a pointer located at distance R from the central rotating shaft. For the spring used, mass M will rotate at radius R for only one rotation period T. Figure 16-2(a) shows the system rotating at the period necessary to rotate at radius R. The period T will be measured for a given R and M. Equation 4 allows a determination of v, and using that value in Equation 3 allows determination of F. This will be referred to as F_{theo} for the theoretical value of the force.

The force the spring exerts on the bob when it is rotating at distance R depends on the amount the spring is stretched under those conditions. This force can be measured by determining the force needed to stretch the spring the same amount when the apparatus is not rotating. Figure 16-2(b) shows a string attached to the other side of the bob with slotted masses applied over a pulley. The weight of the total mass needed to stretch the spring until the tip of the bob is aligned with the pointer is the experimental value of the centripetal force F. This will be referred to as F_{exp}.

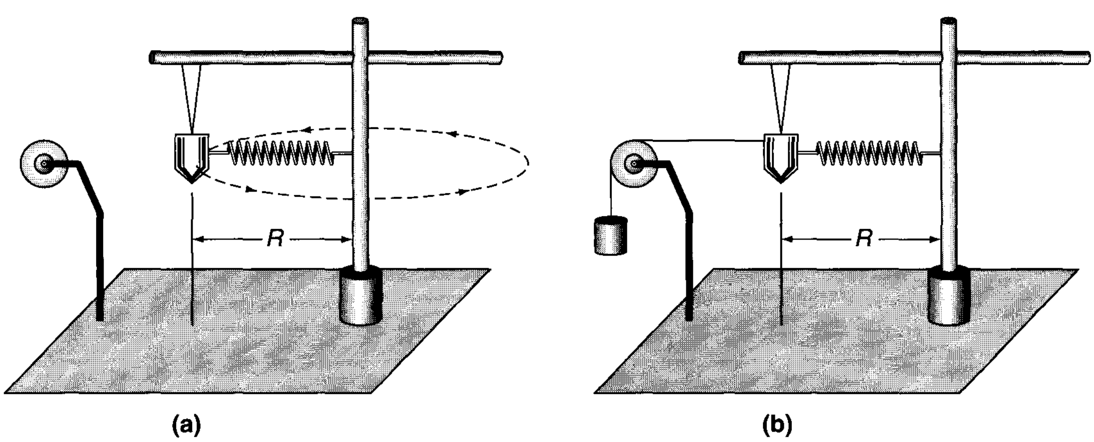

Figure 16-2 (a) Centripetal force apparatus rotating. (b) Determination of the centripetal force by measuring the force needed to stretch the spring under static conditions.

EXPERIMENTAL PROCEDURE

1. Detach the bob from its support strings, remove the spring, and determine its mass. Record this value as m_b in the Data Table.
2. Hang the bob from the cross arm by its support strings (Figure 16-3). Adjust the position of the pointer to its closest position to the rotating shaft for the minimum value of R. Loosen the screw holding the cross arm in the rotating shaft and adjust its position until the tip of the bob is precisely above the tip of the pointer. The tip of the bob should be about 1 mm above the pointer. Measure the distance from the center of the pointer to the center of the rotating shaft. Record this value as R in the Data Table.
3. Attach the spring to the bob and to the rotating shaft. Rotate the system as shown in Figure 16-2(a) by twirling the rotating shaft between the thumb and first finger of your hand. The bob will pass over the pointer at radius R for only one rotation period T. Continue to rotate the apparatus by hand while keeping the rotation speed as constant as possible, and at the same time ensuring that the bob passes over the pointer on each rotation. At this rotation rate measure the time for 25 complete revolutions of the bob and record it in the Data Table as Time 1. Repeat this process two more times, recording the two other measurements of the time for 25 revolutions as Time 2 and Time 3. Record in the Data Table the value of the rotating mass as m_b for this part of the procedure.
4. With the system not rotating, measure directly the centripetal force by attaching a string to the side of the bob opposite the spring. Apply slotted weights over the pulley as shown in Figure 16-2(b) until the tip of the bob is just above the tip of the pointer. Let m_a stand for the total mass needed to stretch the spring by the proper amount. Record in the Data Table the value of m_a needed to stretch the spring to the pointer at position R.
5. Repeat Steps 3 and 4 above using the same R but using two other values of rotating mass. First, add a 0.050 kg slotted mass to the bob. Then remove the 0.050 kg mass and add a 0.100 kg slotted mass. In each case, place the slotted mass with the open end pointed outward, and secure it with the knurled nut on the bob. Record the results of the measurements for rotating mass values of $m_b + 0.050$ kg and $m_b + 0.100$ kg in the Data Table.
6. Perform the measurements with the rotating mass m_{rot} again equal to m_b, but use three new values of R differing by about 1 cm. Each time R is to be changed, remove the spring from the bob, and position the pointer 1 cm further from the rotating shaft. Then adjust the cross arm so that the bob is above the pointer. Perform the measurements of Steps 2–4 for the three values of R and record all results in the Data Table.

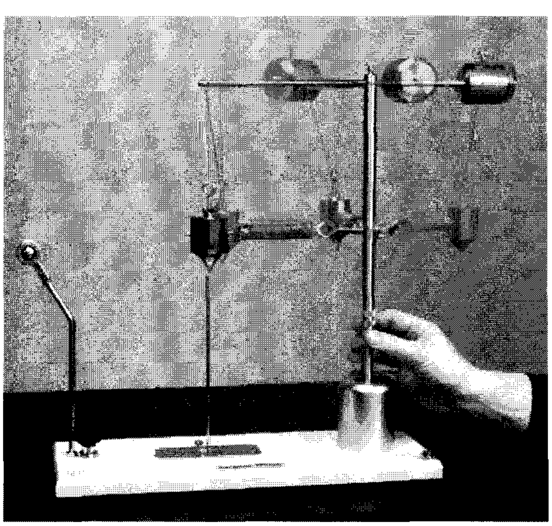

Figure 16-3 Centripetal force apparatus. (Photo by Sargent-Welch Scientific Co.)

CALCULATIONS

1. Calculate the mean of the three trials of the time for 25 complete revolutions, and record it in the Calculations Table as $\overline{Time}$. Divide the value of $\overline{Time}$ by 25, and record the result in the Calculations Table as the period T.
2. Use Equation 4 to calculate v from the measured values of R and T, and record the results in the Calculations Table.
3. Use Equation 3 to calculate the theoretical value for the centripetal force from the values of M, v, and R. Record the results in the Calculations Table as F_{theo}.
4. Calculate the experimental value for the centripetal force as $m_a g$ from the values of m_a. Use a value of 9.80 m/s^2 for g. Record the results in the Calculations Table as F_{exp}.
5. Calculate the percentage difference between the values of F_{theo} and F_{exp}. Record the results in the Calculations Table.

Name

Section

Date

LABORATORY 16 *Centripetal Acceleration of an Object in Circular Motion*

PRE-LABORATORY ASSIGNMENT

1. If a particle moves in a circle of radius R at constant speed v its acceleration is (a) directed toward the center of the circle (b) equal to v^2/R (c) because the direction of the velocity vector changes continuously (d) all of the above are true.

2. If a particle moves in a circle of radius $R = 1.35\,\text{m}$ at a constant speed of $v = 6.70\,\text{m/s}$, what is the magnitude and direction of its centripetal acceleration?

3. If the mass of the particle in Question 2 is 0.350 kg, what is the magnitude and direction of the centripetal force on it? Show your work.

4. A 0.500 kg particle moves in a circle of radius $R = 0.150\,\text{m}$ at constant speed. The time for 20 complete revolutions is 31.7 s. What is the period T of the motion? What is the speed of the particle? Show your work.

5. What is the centripetal acceleration of the particle in Question 4? What is the centripetal force on the particle? Show your work.

6. For the apparatus used in this laboratory the centripetal force is the same for a fixed radius R of rotation. Why is that statement true for this apparatus? (*Hint*—What provides the centripetal force on the rotating mass for this apparatus?)

7. A mass of 0.450 kg rotates at constant speed with a period of 1.45 s at a radius R of 0.140 m in the apparatus used in this laboratory. What is the rotation period for a mass of 0.550 kg at the same radius? Show your work.

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LABORATORY REPORT

Data Table (m_b = ____________ kg)

m_{rot} (kg)	R (m)	*Time 1* (s)	*Time 2* (s)	*Time 3* (s)	m_a (kg)

Calculations Table

m_{rot} (kg)	R (m)	$\overline{Time}$ (s)	T (s)	v (m/s)	F_{theo} (N)	F_{exp} (N)	% *Diff*

SAMPLE CALCULATIONS

1. $\overline{Time} =$
2. $T = \overline{Time}/25 =$
3. $v = (2\pi R)/T =$
4. $F_{theo} = mv^2/R =$
5. $F_{exp} = m_a g =$
6. % Diff =

QUESTIONS

1. Do your results confirm the theoretical relationship for the centripetal acceleration given by $F = Mv^2/R$? Consider the agreement between F_{theo} and F_{exp} to answer this question. Explain your reasoning.

2. Because the centripetal force is provided by a spring for this apparatus, the centripetal force at a given distance R is fixed by the spring constant of the spring. Therefore, Mv^2 should be constant for a given radius R. Calculate the quantity Mv^2 for the four data points taken at the same radius R. Describe the agreement of those values.

3. Equation 3 can be written in the form $v^2 = (1/M)FR$. For a constant value of M this would imply that the quantity v^2 should be proportional to the quantity FR with the reciprocal of the mass as the constant of proportionality. For your data points with the same mass, perform a linear least squares fit with v^2 as the vertical axis and FR as the horizontal axis. Use the values of F_{exp} to calculate FR. Compare the slope of the fit to the reciprocal of the mass. Record the correlation coefficient r.

4. Suppose that a spring with a larger spring constant was used in this same apparatus. If a given mass were rotated at the same radius at which it had been rotated with the original spring, would the new period of rotation using the new spring be greater, or would it be less than the period of rotation using the original spring? Explain your reasoning.