

Measurement of Density

OBJECTIVES

- Determine the mass, length, and diameter of three cylinders of different metals.
- Calculate the density of the cylinders and compare with the accepted values of the density of the metals.
- Determine the uncertainty in the value of the calculated density caused by the uncertainties in the measured mass, length, and diameter.

EQUIPMENT LIST

- Three solid cylinders of different metals (aluminum, brass, and iron)
- Vernier calipers
- Laboratory balance and calibrated masses

THEORY

The most general definition of **density** is mass per unit volume. Density can vary throughout the body if the mass is not distributed uniformly. If the mass in an object is distributed uniformly throughout the object, the density ρ is defined as the total mass M divided by the total volume V of the object. In equation form this is

$$\rho = \frac{M}{V} \quad (\text{Eq. 1})$$

For a cylinder the volume is given by

$$V = \frac{\pi d^2 L}{4} \quad (\text{Eq. 2})$$

where d is the cylinder diameter, and L is its length. Using Equation 2 in Equation 1 gives

$$\rho = \frac{4M}{\pi d^2 L} \quad (\text{Eq. 3})$$

We will determine the quantities M , d , and L by measuring each of them four times and calculating the mean and standard error for each quantity. Using the mean of each measured quantity in Equation 3 leads to the best value for the measured density ρ .

An important question in experimental physics is how the uncertainty in a quantity calculated from other measured quantities is related to the uncertainty in those measured quantities. For this laboratory, the uncertainty in the density (standard error) is related to the standard errors in the mass, length, and diameter by:

$$\alpha_\rho = \rho \sqrt{\left(\frac{\alpha_M}{M}\right)^2 + \left(\frac{\alpha_L}{L}\right)^2 + 4\left(\frac{\alpha_d}{d}\right)^2} \quad (\text{Eq. 4})$$

The form of this equation is stated here without proof, but it can be derived from the relationship between the measured quantities and the density described by Equation 3.

We determine the mass of the cylinders with a laboratory balance, which balances the weight of an unknown mass m against the weight of a known mass m_k . Although the balance is between two forces (the weight of the masses), the scales can be calibrated in terms of mass assuming that the force per unit mass is the same for both the known and unknown mass. The unknown mass on a pan at the left is balanced against the sum of all the known masses placed on the right pan plus the mass equivalent of the permanent sliding mass on the beam. Figure 2-1 shows a picture of a Harvard Trip balance, which has a calibrated beam along which a permanent sliding mass can be moved in units of 0.1 gram up to 10 grams.

The length and diameter of the metal cylinder will be measured with a vernier caliper. A **caliper** is actually any device used to determine thickness, the diameter of an object, or the distance between two surfaces. Often calipers are in the form of two legs fastened together with a rivet, so they can pivot about the fastened point. The vernier caliper used in this laboratory consists of a fixed rule that contains one jaw, and a second jaw with a vernier scale that slides along the fixed rule scale as shown in Figure 2-2. **Vernier** is the name given to any scale that aids in interpolating between marked divisions.

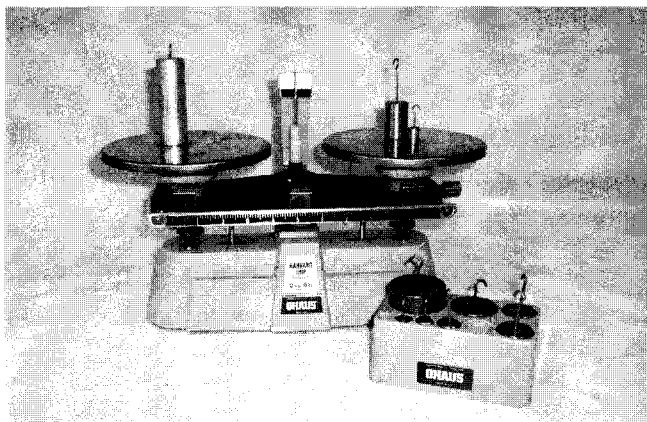


Figure 2-1 Harvard Trip balance.

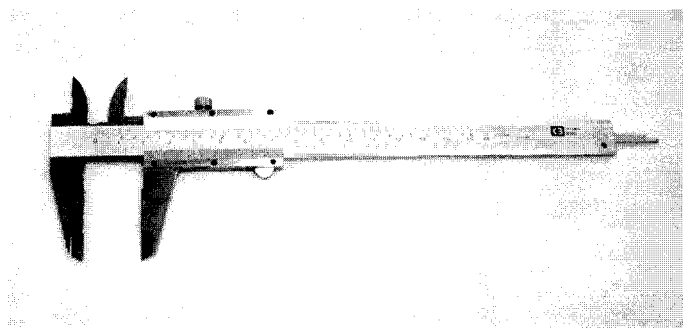


Figure 2-2 Vernier calipers.

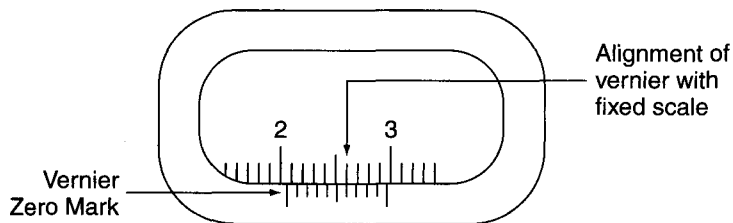


Figure 2-3 Illustration of vernier caliper reading of 2.06 cm.

The caliper has marked on the main scale major divisions of 1 cm for which there are both a mark and a number. On the main scale are also marked 10 divisions, each 1 mm apart between the 1 cm divisions. The 1 mm marks are not labeled with a number. This vernier is marked with a scale that, when aligned with different marks on the fixed rule scale, allows interpolation between the 1 mm marks on the fixed scale to 0.1 mm accurately. A vernier caliper can measure distances accurately to the nearest 0.01 cm.

A measurement is made by closing the jaws on some object and noting the position of the zero mark on the vernier and which one of the vernier marks is aligned with some mark on the fixed rule scale. This is illustrated in Figure 2-3. The position of the zero mark of the vernier scale gives the first two significant figures (2.0 cm in Figure 2-3). We derive the interpolation between 2.0 cm and 2.1 cm for this case from the fact that the sixth mark beyond the vernier zero is best aligned with a mark on the fixed rule scale. The reading in this example is 2.06 cm.

Before making any measurements, determine whether or not the vernier calipers read zero when the jaws are closed. If the calipers do not read zero when the jaws are closed, they are said to have a **zero error**. A correction is necessary for each measurement performed with the calipers. If the vernier zero is to the right of the fixed scale zero when the jaws are closed, the zero error is positive. Note the mark on the vernier scale that is aligned with the fixed scale, and subtract that number of units of 0.01 cm from each measurement. For example, if the third mark to the right of the vernier zero is aligned with the fixed scale when the jaws are closed, then each measurement should have 0.03 cm subtracted from it. If the vernier zero is to the left of the fixed scale zero, then the zero error is negative. In that case, find which vernier mark is aligned with the fixed scale. Then determine how far to the left of the 10 mark on the vernier scale the alignment occurs. For example, if the alignment occurs at the 7 mark on the vernier scale, you will add 0.03 cm to the reading.

EXPERIMENTAL PROCEDURE

1. Zero the laboratory balance according to directions given by your laboratory instructor.
2. Use the laboratory balance and calibrated masses to determine the mass of each of the three cylinders. Make four independent measurements for each of the cylinders and record the results in the Data Table.
3. Make four separate readings of the zero correction for the vernier calipers. Record the four values in the Data Table. Record the zero correction as positive if the vernier zero is to the right of the fixed scale zero and record it as negative if the vernier zero is to the left of the fixed scale zero.
4. Use the vernier calipers to measure the lengths of the three cylinders. Make four separate trials of the measurement of the length of each cylinder. Measure the length at different places on each cylinder for the four trials to sample any variation in length of the cylinders. Record the results in the Data Table.
5. Use the vernier calipers to measure the diameters of the three cylinders. Make four separate trials of the measurement of the diameter of each cylinder. Measure the diameter at four different positions along the length of the cylinders to sample any variation in diameter of the cylinders. Record the results in the Data Table.

CALCULATIONS

1. Calculate the mean \bar{M} , the standard deviation σ_{n-1} , and the standard error α_M for the four measurements of the mass of each cylinder and record the results in the Data Table. For this and all subsequent calculations keep *one significant figure* only for all standard errors, and then keep the number of *decimal places* in the mean that coincides with the *decimal place* of the standard error.
2. Determine the measured length and diameter for each trial by making the appropriate zero correction to each measurement and then calculating the means \bar{L} and \bar{d} , the standard deviations, and the standard errors α_L and α_d for each cylinder. Record the results in the Data Table.
3. Use Equation 3 to calculate the density ρ of each of the cylinders. Use the mean values for the mass, diameter, and length. Use Equation 4 to calculate the standard error of the density. Record the results in the Data Table.
4. For purposes of this laboratory, assume that the density of aluminum is 2.70 gram/cm³, the density of brass is 8.40 gram/cm³, and the density of iron is 7.85 gram/cm³. Calculate the percentage error in your results for the density of each of these metals.

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LABORATORY 2 *Measurement of Density***PRE-LABORATORY ASSIGNMENT**

1. A cylinder has a length of 3.23 cm, a diameter of 1.75 cm, and a mass of 65.3 grams. What is the density of the cylinder? Based on its density, of what kind of material might it be made? Material is likely to be: _____ (Show your work.)

2. Figure 2-4 shows a vernier caliper scale set to a particular reading. What is the reading of the scale?
Reading = _____ cm

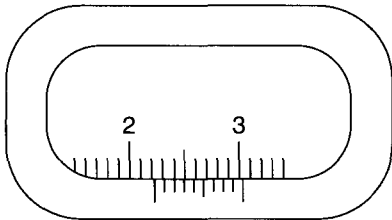


Figure 2-4 Example of a reading of a vernier caliper.

3. The caliper in Figure 2-5 has its jaws closed. If the caliper has a zero error, what is its value? Is it positive or negative? Error = _____ cm

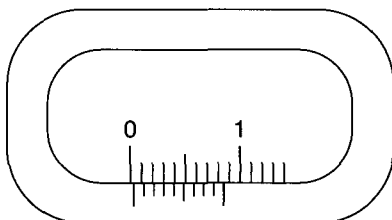


Figure 2-5 Vernier caliper with its jaws closed. Does it have a zero error?

4. A series of four measurements of the mass, length, and diameter are made of a cylinder. The results of these measurements are:

Mass—20.6, 20.5, 20.6, and 20.4 grams

Length—2.68, 2.67, 2.65, and 2.69 cm

Diameter—1.07, 1.05, 1.06, and 1.05 cm

Find the mean, standard deviation, and standard error for each of the measured quantities and tabulate them below. Keep only *one* significant figure in each standard error and then keep *decimal places* in the mean to coincide with the standard error.

$$\bar{M} = \underline{\hspace{2cm}} \quad \sigma_{n-1} = \underline{\hspace{2cm}} \quad \alpha_M = \underline{\hspace{2cm}}$$

$$\bar{L} = \underline{\hspace{2cm}} \quad \sigma_{n-1} = \underline{\hspace{2cm}} \quad \alpha_L = \underline{\hspace{2cm}}$$

$$\bar{d} = \underline{\hspace{2cm}} \quad \sigma_{n-1} = \underline{\hspace{2cm}} \quad \alpha_d = \underline{\hspace{2cm}}$$

Calculate the density and the standard error of the density using Equations 3 and 4. Keep only *one* significant figure in the standard error and then keep *decimal places* in the density to coincide with the standard error.

$$\rho = \underline{\hspace{2cm}} \quad \alpha_\rho = \underline{\hspace{2cm}}$$

5. Because α_ρ has only one digit, it determines the place of the least significant digit kept in the calculation of the density. From that information, how many significant figures are there in the density for the above calculation? State clearly the reasoning for your answer.

Lab Partners _____



LABORATORY 2 *Measurement of Density*

LABORATORY REPORT

Mass Data and Calculations Table

	M_1 (kg)	M_2 (kg)	M_3 (kg)	M_4 (kg)	\bar{M} (kg)	σ_{n-1} (kg)	α_M (kg)
Aluminum							
Brass							
Iron							

Zero Reading of the calipers _____

Length Data and Calculations Table

	L_1 (m)	L_2 (m)	L_3 (m)	L_4 (m)	\bar{L} (m)	σ_{n-1} (m)	α_L (m)
Aluminum							
Brass							
Iron							

Diameter Data and Calculations Table

	d_1 (m)	d_2 (m)	d_3 (m)	d_4 (m)	\bar{d} (m)	σ_{n-1} (m)	α_d (m)
Aluminum							
Brass							
Iron							

3. Consider the value of the standard error α_p as an indication of the precision of your measurements. Express the standard error as a percentage of the measured value of the density, and relate it to the accuracy of each of your measurements.

4. Considering your answers to Question 3, is there evidence for a systematic error in any of your measurements of the density of any of the metals? State clearly your evidence either for or against the presence of a systematic error.

5. For the same percentage error in each of the three quantities, mass, diameter, and length, which would contribute the most to the error in the density? (Hint—Consider the form of Equation 4.)