Force Table and Vector Addition of Forces

OBJECTIVES

- ❑ Demonstrate the addition of several vectors to form a resultant vector using a force table.
- ❑ Demonstrate the relationship between the resultant of several vectors and the equilibrant of those vectors.
- ❑ Illustrate and practice graphical and analytical solutions for the addition of vectors.

EQUIPMENT LIST

- Force table with pulleys, ring, and string
- Mass holders and slotted masses
- Protractor and compass

THEORY

Physical quantities that can be completely specified by magnitude only are called **scalars**. Examples of scalars include temperature, volume, mass, and time intervals. Some physical quantities have both magnitude and direction. These are called **vectors**. Examples of vector quantities include spatial displacement, velocity, and force.

Consider the case of several forces with different magnitudes and directions that act at the same point. The single force, which is equivalent in its effect to the effect produced by the several applied forces, is called the **resultant force**. This resultant force can be found theoretically by a special addition process known as **vector addition**.

One process of vector addition is by graphical techniques. Figure 3-1(a) shows the case of two vectors, $\mathbf{F_1}$ of magnitude 20.0 N, and $\mathbf{F_2}$ of magnitude 30.0 N. A scale of 1.00 cm = 10.0 N is used, and these vectors are shown as 2.00 cm and 3.00 cm in length, respectively. The forces are assumed to act at the same point, but 60° different in direction as shown. Figure 3-1(b) shows the graphical addition process called the **parallelogram method**. Two lines are constructed, each one parallel to one of the vectors having the length of that vector as shown. The resultant $\mathbf{F_R}$ of the vector addition of $\mathbf{F_1}$ and $\mathbf{F_2}$ is found by constructing the straight line from the point at the tails of the two vectors to the opposite corner of the parallelogram formed by the original vectors and the constructed lines. A measurement of the length of $\mathbf{F_R}$ in Figure 3-1(b) shows it to be 4.35 cm in length, and a measurement of the angle between $\mathbf{F_R}$ and $\mathbf{F_1}$ shows it to be about 37°. Because the scale is 1.00 cm = 10.0 N, the value of the resultant $\mathbf{F_R}$ is 43.5 N, and it acts in a direction 37° with respect to the direction of $\mathbf{F_1}$.

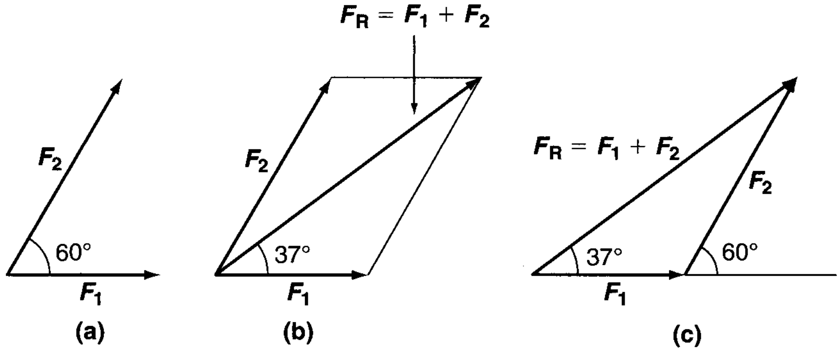


Figure 3-1 Illustration of the parallelogram and triangle addition of vectors.

In the graphical vector addition process known as the **polygon method** one of the vectors is first drawn to scale. Each successive vector to be added is drawn with its tail starting at the head of the preceding vector. The resultant vector is the vector drawn from the tail of the first arrow to the head of the last arrow. Figure 3-1(c) shows this process for the case of only two vectors (for which the polygon method is the triangle method). The second vector, $\mathbf{F_2}$, must be drawn at the proper angle relative to $\mathbf{F_1}$ by extending a line in the direction of $\mathbf{F_1}$ and constructing $\mathbf{F_2}$ relative to that line. In Figure 3-1(c) the length of $\mathbf{F_R}$ is 4.35 cm corresponding to 43.5 N, and it acts at 37° with respect to $\mathbf{F_1}$.

The polygon method is illustrated for the case of three vectors in Figure 3-2. Vector $\mathbf{F_1}$ is drawn, $\mathbf{F_2}$ is drawn at the proper angle α relative to $\mathbf{F_1}$, and $\mathbf{F_3}$ is drawn at the proper angle β relative to $\mathbf{F_2}$. The resultant $\mathbf{F_R}$ is the vector connecting the tail of $\mathbf{F_1}$ and the head of $\mathbf{F_3}$.

The analytical process of vector addition uses trigonometry to express each vector in terms of its components projected on the axes of a rectangular coordinate system. Figure 3-3 shows a vector, a coordinate system superimposed on the vector, and the components $|F|\cos\theta$ and $|F|\sin\theta$ into which the vector is resolved. When the analytical process for multiple vectors is used, each vector is resolved into components in that manner. The components along each axis are then added algebraically to produce the net components of the resultant vector along each axis. Those components are at right angles, and the magnitude of the resultant can be found from the Pythagorean theorem. The case of three vectors, $\mathbf{F_1}$, $\mathbf{F_2}$, and $\mathbf{F_3}$, is shown in Figure 3-4.

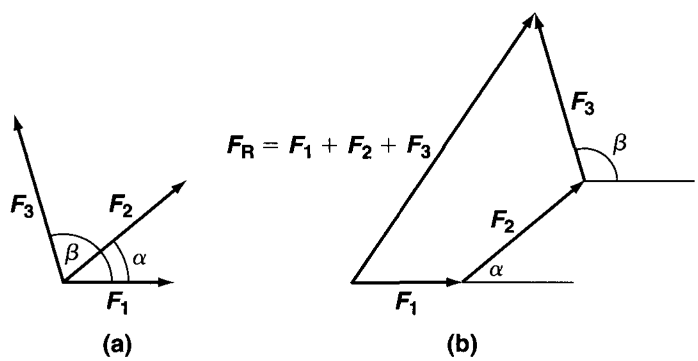


Figure 3-2 Illustration of the polygon method for vector addition.

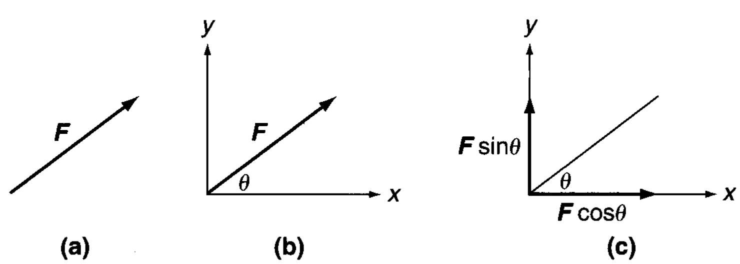


Figure 3-3 Illustration of analytical resolution of a vector.

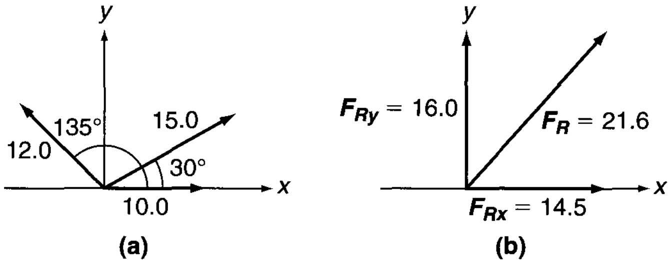


Figure 3-4 Illustration of the analytical addition of vectors.

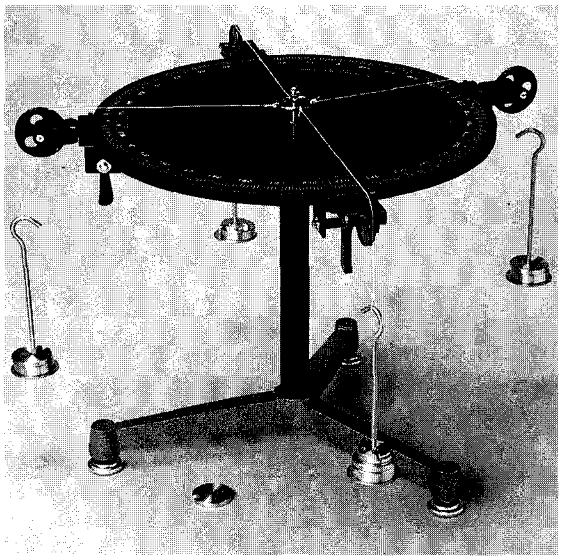


Figure 3-5 Force Table. (Photo courtesy of Sargent-Welch Scientific Company)

Taking the algebraic sum of each of the components of the three vectors and combining the components to find the resultant and its direction leads to the following:

$$F_{Rx} = F_{1x} + F_{2x} + F_{3x} = 10.0\cos(0°) + 15.0\cos(30°) + 12.0\cos(135°) = 14.5$$

$$F_{Ry} = F_{1y} + F_{2y} + F_{3y} = 10.0\sin(0°) + 15.0\sin(30°) + 12.0\sin(135°) = 16.0$$

$$|\boldsymbol{F_R}| = F_R = \sqrt{(F_{Rx})^2 + (F_{Ry})^2} = \sqrt{(14.5)^2 + (16.0)^2} = 21.6$$

$$\theta = \arctan(F_{Ry}/F_{Rx}) = \arctan(16.0/14.5) = \text{arc tan}\ (1.10) = 47.7°$$

The force table (Figure 3-5) provides a force from the gravitational attraction on masses attached to a ring by a string passing over a pulley. Each force is applied over a separate pulley, and the pulley positions can be adjusted to any desired position around a circular plate. Experimentally the applied forces are balanced by the application of a single force that is equal to the magnitude of the resultant of the applied forces and acts opposite of the resultant. This balancing force (called the **equilibrant**) is what is determined by the measurements. The resultant is the same magnitude as the equilibrant and 180° different in direction.

EXPERIMENTAL PROCEDURE

Part 1. Two Applied Forces

1. Place a pulley at the 20.0° mark on the force table and place a total of 0.100 kg (including the mass holder) on the end of the string. Calculate the magnitude of the force (in N) produced by the mass.

Assume that $g = 9.80\ m/s^2$. Assume three significant figures for this and for all other calculations of force. Record the value of this force as F_1 in Data Table 1.

2. Place a second pulley at the 90.0° mark on the force table and place a total of 0.200 kg on the end of the string. Calculate the force produced and record as F_2 in Data Table 1.
3. Determine by trial and error the magnitude of mass needed and the angle at which it must be located for the ring to be centered on the force table. Jiggle the ring slightly to be sure that this equilibrium condition is met. Attach all strings to the ring so that they are directed along a line passing through the center of the ring. All the forces will then act through the point at the center of the table. Record this value of mass in Data Table 1 in the row labeled Equilibrant F_{E1}.
4. Calculate the force produced (mg) on the experimentally determined mass. Record the magnitude and direction of this equilibrant force F_{E1} in Data Table 1.
5. The resultant F_{R1} is equal in magnitude to F_{E1}, and its direction is 180° from F_{E1}. Record the magnitude of the force F_{R1}, the mass equivalent of this force, and the direction of the force in Data Table 1 in the row labeled Resultant F_{R1}.

Part 2. Three Applied Forces

1. Place a pulley at 30.0° with 0.150 kg on it, one at 100.0° with 0.200 kg on it, and one at 145.0° with 0.100 kg on it.
2. Calculate the force produced by those masses and record them as F_3, F_4, and F_5 in Data Table 2.
3. Determine the equilibrant force and the resultant force by following a procedure like that in Part 1, Steps 3 through 5 above. Record the magnitudes of the forces, the associated values of mass, and the directions in Data Table 2 in the rows labeled F_{E2} and F_{R2}.

CALCULATIONS

Part 1. Two Applied Forces

1. Find the resultant of these two applied forces by scaled graphical construction using the parallelogram method. Use a ruler and protractor to construct vectors with scaled length and direction that represent F_1 and F_2. A convenient scale might be 1.00 cm = 0.100 N. All directions are given relative to the force table. Account for this in the graphical construction to ensure the proper angle of one vector to another. Determine the magnitude and direction of the resultant from your graphical solution and record them in the appropriate section of Calculations Table 1.
2. Use trigonometry to calculate the components of F_1 and F_2 and record them in the analytical solution portion of Calculations Table 1. Add the components algebraically and determine the magnitude of the resultant by the Pythagorean theorem. Determine the angle of the resultant from the arc tan of the components. Record those results in Calculations Table 1.
3. Calculate the percentage error of the magnitude of the experimental value of F_R compared to the analytical solution for F_R. Also calculate the percentage error of the magnitude of the graphical solution for F_R compared to the analytical solution for F_R. For each of those comparisons, also calculate the magnitude of the difference in the angle. Record all values in Calculations Table 1.

Part 2. Three Applied Forces

1. Use the polygon scaled graphical construction method to find the resultant of the three applied forces. Determine the magnitude and direction of the resultant from your graphical solution and record them in Calculations Table 2.
2. Use trigonometry to calculate the components of all three forces, the components of the resultant, and the magnitude and direction of the resultant, and record them all in Calculations Table 2.
3. Make the same error calculations for this problem as described in Step 3 of Part 1 above. Record the values in Calculations Table 2.

Name Section Date

LABORATORY 3 *Force Table and Vector Addition of Forces*

PRE-LABORATORY ASSIGNMENT

1. Scalars are physical quantities that can be completely specified by their ____________________.

2. A vector quantity is one that has both ____________________ and ____________________.

3. Classify each of the following physical quantities as vectors or scalars:
   (a) Volume ________
   (b) Force ________
   (c) Density ________
   (d) Velocity ________
   (e) Acceleration ________

Answer Questions 4–7 with reference to Figure 3-6 below.

4. If $\mathbf{F_1}$ stands for a force vector of magnitude 30.0 N and $\mathbf{F_2}$ stands for a force vector of magnitude 40.0 N acting in the directions shown in Figure 3-6, what are the magnitude and direction of the resultant obtained by the vector addition of these two vectors using the analytical method? Show your work.

   Magnitude = ________N Direction(relative to x axis) = ________degrees

5. What is the equilibrant force that would be needed to compensate for the resultant force of the vectors $\mathbf{F_1}$ and $\mathbf{F_2}$ that you calculated in Question 4?

   Magnitude = ________N Direction(relative to x axis) = ________degrees

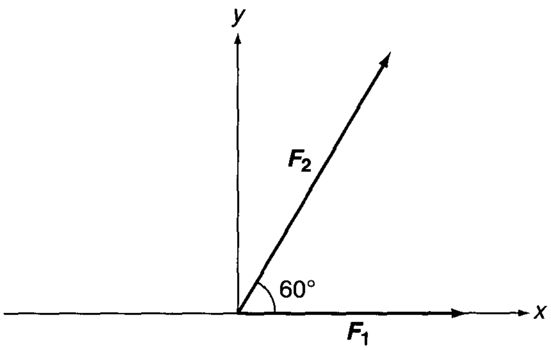


Figure 3-6 Addition of two force vectors.

6. Figure 3-6 has been constructed to scale with 1.00 cm = 10.0 N. Use the parallelogram graphical method to construct (on Figure 3-6) the resultant vector $\mathbf{F_R}$ for the addition of $\mathbf{F_1}$ and $\mathbf{F_2}$. Measure the length of the resultant vector and record it below. State the force represented by this length. Measure with a protractor the angle that the resultant makes with the x axis.

   Resultant vector length = ________cm

   Force represented by this length = ________N

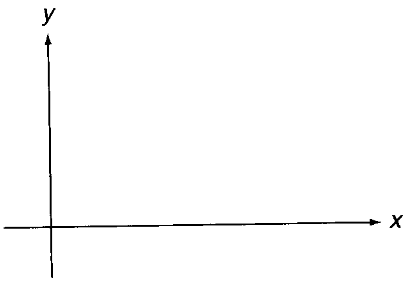
   Direction of resultant relative to x axis = ________degrees

7. Use the polygon method of vector addition to construct on the axes below a graphical solution to the problem in Figure 3-6. Use the scale 1.00 cm = 10.0 N.

   Resultant vector length = ________cm

   Force represented by this length = ________N

   Direction of resultant relative to x axis = ________degrees



Name | Section | Date

Lab Partners

LABORATORY 3 *Force Table and Vector Addition of Forces*

LABORATORY REPORT

Data Table 1

Force	*Mass* (kg)	*Force* (N)	*Direction*
F_1	0.100		20.0°
F_2	0.200		90.0°
Equilibrant F_{E1}			
Resultant F_{R1}			

Data Table 2

Force	*Mass* (kg)	*Force* (N)	*Direction*
F_3	0.150		30.0°
F_4	0.200		100.0°
F_5	0.100		145.0°
Equilibrant F_{E2}			
Resultant F_{R2}			

Calculations Table 1

Graphical Solution			
Force	*Mass* (kg)	*Force* (N)	*Direction*
F_1	0.100		20.0°
F_2	0.200		90.0°
Resultant F_{R1}			

Analytical Solution					
Force	*Mass* (kg)	*Force* (N)	*Direction*	*x-component*	*y-component*
F_1	0.100		20.0°		
F_2	0.200		90.0°		
Resultant F_{R1}					

PART 1. ERROR CALCULATIONS

Percent Error magnitude Experimental compared to Analytical = ________%

Percent Error magnitude Graphical compared to Analytical = ________%

Absolute Error in angle Experimental compared to Analytical = ________degrees

Absolute Error in angle Graphical compared to Analytical = ________degrees

Calculations Table 2

Graphical Solution			
Force	*Mass* (kg)	*Force* (N)	*Direction*
F_3	0.150		30.0°
F_4	0.200		100.0°
F_5	0.100		145.0°
Resultant F_{R2}			

Analytical Solution					
Force	*Mass* (kg)	*Force* (N)	*Direction*	*x-component*	*y-component*
F_3	0.150		30.0°		
F_4	0.200		100.0°		
F_5	0.100		145.0°		
Resultant F_{R2}					

PART 2. ERROR CALCULATIONS

Percent Error magnitude Experimental compared to Analytical = ________%

Percent Error magnitude Graphical compared to Analytical = ________%

Absolute Error in angle Experimental compared to Analytical = ________degrees

Absolute Error in angle Graphical compared to Analytical = ________degrees

SAMPLE CALCULATIONS

1. $F = mg =$
2. $m = \frac{F}{g} =$
3. Direction F_E opposite F_R so direction F_R = direction $F_E - 180° =$
4. $F_{1x} = F_1 \cos(20°) =$
5. $F_{1y} = F_1 \sin(20°) =$
6. $F_{R1} = \sqrt{(F_{Rx})^2 + (F_{Ry})^2} =$
7. $\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) =$
8. $\% \text{ Error Exp} = \frac{|\text{Experimental} - \text{Analytical}|}{\text{Analytical}} \times 100\% =$
9. Absolute Err $= \theta(\text{exp}) - \theta(\text{analytical}) =$

QUESTIONS

1. To determine the force acting on each mass it was assumed that $g = 9.80\ \text{m/sec}^2$. The value of g at the place where the experiment is performed may be slightly different from that value. State what effect (if any) it would have on the *percentage error* calculated for the comparisons. To test your answer to the question, leave g as a symbol in the calculation of the percentage error.

2. Two forces are applied to the ring of a force table, one at an angle of 20.0°, and the other at an angle of 80.0°. Regardless of the magnitudes of the forces, choose the correct response below.

   The equilibrant will be in the (a) first quadrant (b) second quadrant (c) third quadrant (d) fourth quadrant (e) cannot tell which quadrant from the available information.

The resultant will be in the (a) first quadrant (b) second quadrant (c) third quadrant (d) fourth quadrant (e) cannot tell which quadrant from the available information.

3. Two forces, one of magnitude 2 N and the other of magnitude 3 N, are applied to the ring of a force table. The directions of both forces are unknown. Which *best* describes the limitations on R, the resultant? Explain carefully the basis for your answer.

(a) $R \leq 5$ N **(b)** 2 N $\leq R \leq$ 3 N **(c)** $R \geq 3$ N **(d)** 1 N $\leq R \leq$ 5 N **(e)** $R \leq 2$ N.

4. Suppose the same masses are used for a force table experiment as were used in Part 1, but each pulley is moved 180° so that the 0.100 kg mass acts at 200°, and the 0.200 kg mass acts at 270°. What is the magnitude of the resultant in this case? How does it compare to the resultant in Part 1?

5. Pulleys introduce a possible source of error because of their possible friction. Given that they are a source of error, why are the pulleys used at all? What is the function of the pulleys?