Final Exam – MAT 139 – Fall 2007

True/False questions

Circle the correct answer (True or False) and give a brief justification.

1. True or False: When finding the absolute minimum value that a function attains on some closed interval, we need only inspect those points where the derivative is positive.

2. True or False: The derivative of e^x is e^x.

3. True or False: There are many choices for the anti-derivative of a function $f(x)$.

4. True or False: The area under the graph of a function on some interval (a, b) can be computed with reference to one of its anti-derivatives only evaluated at a and b.

5. True or False: A root of a function's derivative is necessarily a place where the function has an extremum.

6. True or False: One anti-derivative for $f(x) = 1/x$ is $F(x) = \ln(x)$.

7. True or False: There are exactly two functions that have the property that $f(x) = f'(x)$ for all values of x.

8. True or False: The derivative of a function is defined by the limit

$$\lim_{h \longrightarrow 0} \frac{f(x) - f(x+h)}{h}.$$

9. Find the derivatives of the following monomials.

(a) x^5

(b) $\frac{x^{100}}{100}$

(c) $\frac{1}{4}x^{-4}$

(d) $x^{1/5}$

(e) $\pi x^{-3/4}$

10. Find the derivatives of the following functions.

(a) $f(x) = x^3 - x^2 + x - 1$

(b) $g(x) = \sqrt[5]{x} - \sqrt[3]{x}$

(c) $h(x) = \dfrac{x^3 - x}{x^2 + 1}$

(d) $j(x) = (x^2 + 3x + 1) \cdot (x^2 - 3x + 1)$

(e) $k(x) = (x^7 - x^5 + x^3 - x)^4$

11. Find the derivatives of the following functions involving logarithms.

- $\ln(x)$
- $\ln(x^4/(x^2+1))$
- $\log_{10}(x)$

12. Find anti-derivatives for each of the following.

- x^4
- $3x^5 + 2x^3 + x$
- $x^{-1/3}$
- $x^{1/3} + x^{-1/2}$

13. Find the area of the region in the 1st quadrant bounded by the curves $y = x$ and $y = x^4$.

14. Find the area under the curve $y = x^3$, above the x-axis, and between the verical lines $x = 2$ and $x = 4$.

15. Fill in the blanks:

    (a) $f'(a) > 0$ means that $f(x)$ is _____ when $x = a$.

    (b) $f'(a) < 0$ means that $f(x)$ is _____ when $x = a$.

    (c) $f''(a) > 0$ means that $f(x)$ is _____ when $x = a$.

    (d) $f''(a) < 0$ means that $f(x)$ is _____ when $x = a$.

16. A rectangular pen is to have 60 square meters of land for holding pigs. One side of the pen will be formed by the side of a barn so fencing is only required on three sides. What dimensions will minimize the amount of fencing required?

17. Find the 1st, 2nd and 3rd derivatives of the following functions.

(a) $f(x) = x^3$

(b) $g(x) = \sqrt[5]{x}$

(c) $h(x) = e^{3x}$

18. Identify the intervals in which the graph of the function $y = x^4 - 2x^3 - 12x^2 + 5$ is concave up and/or concave down. What are its points of inflection?

19. What is the average value of the function $f(x) = x^3 - x^2$ over the interval $(0, 5)$?

20. Determine each of the following indefinite integrals.

(a) $\int 5x^2 \, \mathrm{d}x \; =$

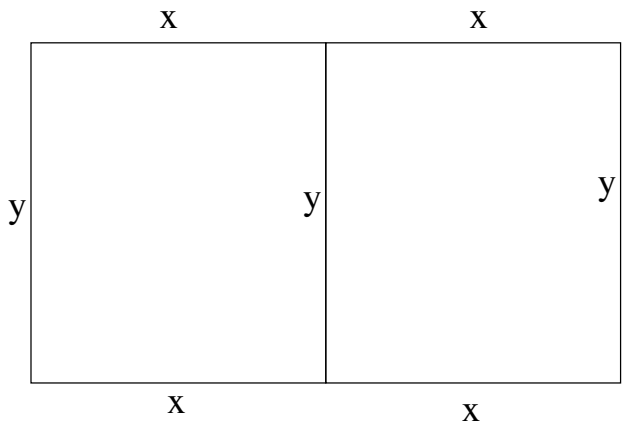
(b) $\int 3x^5 - 2x^3 + 3x \, \mathrm{d}x \; =$

(c) $\int \sqrt{x^3} \, \mathrm{d}x \; =$

(d) $\int \dfrac{2}{x} \, \mathrm{d}x \; =$

(e) $\int e^x \, \mathrm{d}x \; =$

21. A rectangular corral is to be built containing two equal rectangular pens that share a side.



Find the dimensions of the corral that uses the minimum amount of fencing subject to the constraint that the two pens contain a total area of 100 square meters.

22. Draw a careful graph of $y = 2x^3 + 3x^2 - 36x + 1$. Identify any relative extrema and points of inflection.

23. Find the derivatives of each of the following:

(a) $\ln x \cdot e^x$

(b) $\dfrac{\ln(x)}{x}$

(c) e^{x^2-1}

24. Find the elasticity of demand when $p = 5$ if demand for a given commodity is modelled by $q = \dfrac{300}{p^2} - 8$.