MAT 108
ERROR ANALYSIS
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Approximate numbers

When we are working with numbers in the real world, we have several limitations:

1. How we measure physical quantities.
2. How we represent numbers.

Any number determined by a counting process or by definition is an *exact* number. When the value of a variable is an exact number, we use the equal sign to express that relationship. For example, x = 2 states that the value of x is exactly 2.

Any number determined by some method of measurement is an *approximate* number. Sometimes we replace counting or measuring a quantity by *estimating* the number. An estimated number is also an approximate number. Finally, when we round a number, the result is an approximate number. When the value of a variable is an approximate number, we use the approximately equal sign $\approx$, not the equal sign, to express that relationship. For example, if x =2/3, then $x \approx 0.67$.

Representation of numbers

Sometimes we want to change how we represent a number. For example, consider the following number.

46.527415928762

In working with this number, we may not want to keep all of the digits. For example, we might want to keep the number only to two decimal places, i.e., to the hundredths place. There are several standard ways that we can change the representation of a number: truncation, rounding, or always rounding up. In this course, we will use the general rounding process, unless otherwise specified.

1. *truncation (chopping or always rounding down)*
   We discard all of the digits beyond the digit in the last decimal place that we want to keep. Some scientific instruments are calibrated this way. For example, a digital odometer on a car might be set so that it does not add a mile to the total until the full mile has been traveled.

   Example: If we truncate 46.527415928762 to the hundredths place, we get 46.52.

2. *rounding*

When we round a number, we represent that number by the closest number whose last decimal place is the desired place value. For example, suppose we want to round 4.3281 to the hundredths place. Since $4.32 \leq 4.3281 \leq 4.33$, we could reasonably represent 4.3281 by either 4.32 or 4.33. Since 4.3281 is closer to 4.33, we will represent 4.3281 by 4.33, and we will say that we have rounded 4.3281 to 4.33.

To round a number to a specific place value, we first discard all of the digits in places to the right of that place value, replacing them with zeros if they are to the left of the decimal point. We call the digit in the desired place value, the *round-off digit*. If the first digit discarded is less than 5, we leave the round-off digit as it is (round-down or truncate). If the first digit discarded is 5 or more, we increase the round-off digit by 1 (round-up). These are the rules we will use in this course.

Example: If we round 35.764 to the nearest tenth, we get 35.8.

Example: If we round .57243 to four decimal places, we get .5724.

Example: If we round 81.465 to the nearest hundredth, we get 81.47.

Example: If we round 46.527415928762 to the nearest hundredth, we get 46.53.

Example: If we round 23856**.**4 to the nearest thousand, we get 24000 as our representation.

Some people may use the following additional rule, sometimes called the *Rule of Five*: If the first digit discarded is a five followed by zeros or no other digits, then we round up if the test digit is odd and round down if the test digit is even. Then in the example above, 81.465 would round to 81.46, since the round-off digit, 6, is even.

3. *always rounding up*

This rule is used in some applications. For example, the post office rounds up to the nearest ounce when computing postage and the phone company generally rounds up to the nearest minute when timing the length of calls. Also, we sometimes do this when we want to be sure that we have overestimated the value of a measurement or computation, for example, in determining how much material it will take to cover the outside of a box. *We will rarely, if ever, use this method in this course.*

Significant digits

Consider the following story:

> *A mathematician was being shown around a museum by a museum guide. When they came to the dinosaur section, the museum guide said, "This is the oldest dinosaur on display. It's twenty million and three years old." "How can you be so definite about its age?" asked the mathematician. "Well", replied the guide, "they told me that it was twenty million years old when I started to work here three years ago."*

Is the story funny? If so, what is the critical idea that makes it humorous? Are the dinosaurs in the museum exactly 23 million years old or approximately 23 million years old?

A zero can serve two purposes in an approximate number: it can represent a value OR it can be used just as a place holder to properly locate the decimal point. For example, since 3 centimeters and .03 meters represent the same physical length, the only purpose of the zero in the .03 is to properly locate the decimal place; hence the zero is acting only as a place holder. Except for zeros which are used **only** as place holders, all other digits are called *significant digits*.

We can use the following rules in determining the number of significant digits in an approximate number.

1. The first significant digit in a number is the first nonzero digit checking from left to right.

   Example: 5 is the first significant digit in .00564.

2. All nonzero digits are significant.

   Example: 345.28 has five significant digits.

3. Zeros in front of the first nonzero digit are not significant.

   Example: .0004927 has four significant digits.

4. Zeroes between nonzero digits are significant.

   Example: 40500.74 has seven significant digits.

   Example: .003092 has four significant digits.

5. Zeros at the right end of a number with a decimal point are significant.

Example: 5.7300 has five significant digits.

Example: 230**.** has three significant digits.

6. Final zeros in approximate numbers not explicitly containing the decimal point may or may not be significant. For example, 230 may have two significant digits or it may have three significant digits. We are not given enough information to determine which , since we do not know whether the final zero is just a place holder. In some cases, the number may be in a context that will allow us to determine whether or not the final zeros are significant. In general, we will assume that such final zeros are NOT significant unless we are given explicit information to the contrary. Hence, we would assume that 230 has only two significant digits. If we know the zero is significant, then we should express the number using a decimal point, 230**.**, as in the example above. Some numbers, for example 2300, have more than one final zero, and then the situation is even more unclear. 2300 may have two significant digits if neither zero is significant, or it may have three significant digits if the first zero is significant and the second is not, or it may have four significant digits if both zeros are significant. As above, without additional information we would have to assume that 2300 has only two significant digits.

   Unless all final zeros in a number are significant, final zeros that are significant should be written using scientific notation, where the coefficient contains all of the significant digits and only significant digits. For example, without additional information we would assume that 15000 has only two significant digits. This would be expressed in scientific notation by writing the number as 1.5×10^4, where only the 1 and the 5 are significant. If however we assume that the first zero is also significant but the other two zeroes are not significant, then we would express the number as 1.50×10^4 , where by including the zero in the coefficient we are indicating that it is significant. Similarly, 1.500×10^4 would have four significant digits because two significant zeros are included. Finally, 15000. would indicate that the number has five significant digits because we have included the decimal point, and could alternatively be written as 1.5000×10^4.

To round an approximate number to a specific number of significant digits, you count off the number of significant digits, counting from left to right, and then round to the appropriate decimal place.

> *Example:* Round 54.719 to three significant digits. Counting from left to right, the third significant digit is in the tenths place, so we will round the number to the nearest tenth. Hence, our rounded number will be 54.7.

Example: Round 64,592 to two significant digits. 65,000 will be the number rounded to two significant digits.

Measurement and error

Whenever a number is determined by a measurement or estimation process, that number will always be approximate, and hence there will always be measurement *error* associated with that number. There are several ways that we can describe error.

Definition: The difference between a measured value and the exact value of a quantity is called the *absolute error*. Synonyms for the exact value include accepted value, handbook value, and theoretical value. More specifically,

absolute error = measured value – exact value

Usually we do not know the exact value and do not have any "accepted value". There are several ways to handle this.

Option 1: We may take multiple readings and use the average (mean) of those readings as the "exact value".

Option 2: We find a "maximum possible absolute error", which in this course (but perhaps nowhere else in the mathematical or scientific literature) we shall denote by E_{max} . This maximum possible absolute error would be determined from a combination of the measurement(s), knowledge of the measuring instruments, and our experience. In most cases, when there is no known exact value or accepted value, E_{max} will just be referred to as the absolute error. Given this definition,

$$\textit{measured value} - E_{max} \le \textit{exact value} \le \textit{measured value} + E_{max}.$$

We often express this relationship by *measured value* $\pm E_{max}$.

Example: A measurement is given as 85 ± 3. The measured value is 85, and E_{max}, the maximum possible absolute error, is 3. Hence, the exact value could be as low as $85 - 3 = 82$ and as high as $85 + 3 = 88$, so

$$82 \le \text{exact value} \le 88.$$

Example: Suppose we take a patient's temperature and get a reading of 98.3° F. However, we know from past experience that this reading could be off by as much as **.**4° F. This **.**4° F is E_{max}, the maximum possible absolute error. The patient's actual temperature then could be as low as 97.9° F or as high as 98.7° F. I.e.,

$$97.9° \text{ F} \leq \text{ patient's temperature} \leq 98.7° \text{ F},$$

and the measurement could be presented by 98.3 ° F ± .4 ° F, or more simply, 98.3 ± .4 ° F. Note that if a measured quantity has units, then the absolute error will also have units.

When we measure something, even in the best situation, we always need to round to get the last digit of the number. For example, if we are using a ruler to measure a length, we might round to the nearest millimeter, or if we are timing something, we might round to the nearest second. This leads to the following rule involving measurement:

Rule: Whenever a number is given that is based on a measured quantity, unless a more specific description of error is provided, it should be assumed that there may be an error of up to ±0.5 in the place of the last significant digit. Since this error is always present, we will call it the *default error*, meaning that even if no error is specifically mentioned this error will always be there. Note that the default error is the smallest possible absolute error.

Example: Given the approximate number 34**.**240, the last significant digit is in the thousandths place. So, the measurement could be off by up to .5(.001) = .0005. Hence, the default error is ±**.**0005.

Example: Given the approximate number 8300, since we can only assume two significant digits, the last significant digit is in the hundreds place. So, the default error is ±50, since 50 =**.**5(100).

For a given measurement, the default error is the smallest possible absolute error. So, we can interpret that measurement in terms of an interval.

Example: Consider the approximate number 41.73. Since the measurement could be off by up to .5 in the decimal place of the last significant digit, which in this case is the hundredths place, the measurement could be off by up to .005 = .5(.01), and the default error is ±.005. Hence, 41.73 could represent any number in the interval from 41.725 (= 41.73–.005) to 41.735 (= 41.73+.005). Then, $41.725 \leq \textit{exact number} \leq 41.735$. So, we could say that the approximate number 41.73 represents the closed interval [41.725 , 41.735]. A convenient way to represent this interval of values is with the notation 41.73±.005.

Example: Consider the approximate number .00378. In this case the last significant digit is in the hundred-thousandths place. Since .5(.00001)=.000005, the default error is ±.000005, and this approximate number could represent any number from .003775 to .003785, or we could write this as .00378±.000005. So $.003775 \leq \textit{exact number} \leq .003785$. This range can also be represented by the interval of values [.003775 , .003785].

Example: Suppose we are given a measurement 24.7, without any explicit representation of error. Then we assume that the number has as least the default error, which in this case is ± .05, so E_{max} = .05. In this case, $24.65 \le \textit{exact number} \le 24.75$.

Relative and Percentage Error

There are other ways that we can characterize error.

Suppose in using a ruler to measure the radius of a circle, we know that $E_{\max} = 1/16 \text{ inch} = .0625 \text{ inch.}$ How bad is this error? The magnitude of the error depends upon the actual size of the subject being measured. For example, if the radius is very small, say only 0.25 inch, then the error is large. However, if the radius is large , say 25 inches, then this error is not too bad. *Relative error* compares the absolute error to the size of the actual measurement.

Definition: The *relative error* of a measurement is the absolute value of the absolute error divided by the exact value. When the relative error is expressed as a percentage, we call it the *percentage error*.

$$\textit{relative error} = \frac{|\textit{absolute error}|}{\textit{exact value}}.$$

$$\textit{percentage error} = \frac{|\textit{absolute error}|}{\textit{exact value}} \times 100\%.$$

Important note: When we do not have the exact value, we also can not compute the exact absolute error. In such cases, we will use E_{max} as the absolute error and the measurement (or the average of a set of measurements) in place of the exact value, and so approximate the relative error as follows:

$$\textit{relative error} \approx \frac{E_{\max}}{\textit{measured value}}$$

This will be the situation in most applications, since if we know the exact value, there is usually no need to measure.

Example: In our example, if the measurement of the radius is $.25 \pm .0625 \text{ inches}$, Then

$$relative\ error \approx \frac{E_{\max}}{measured\ value} = \frac{.0625}{.25} = 0.25$$

and *percentage error* = 25%.

If, however, the measurement of the radius is $25 \pm .0625$ inches, then

$$relative\ error \approx \frac{E_{\max}}{measured\ value} = \frac{.0625}{25} = .0025$$

and the percentage error is **.**25%.

Precision and accuracy

In describing measurements and approximate numbers, we will use the terms *precision* and *accuracy*.

In most of our work, we will not know the exact value and hence will never know the exact absolute error. For that reason we will use the terms *absolute error* and *maximum possible absolute error* interchangeably.

Precision is a description of absolute error. The smaller the absolute error, the more precise the measurement. In general, the unit of measure of the measurement determines the precision of the measurement. The precision of a measurement is often related to the "precision" of the measuring instrument. (In the sciences, the term *precision* is often used to describe the reproducibility of a measurement.)

Example: Consider the two measurements

$246.3 \pm .4$ and $143.78 \pm .06$.

The second measurement 143.78 is more precise, because its absolute error is smaller.

Example: Consider the two measurements

$846. \pm .5$ and $492.3 \pm .05$.

The measurement 492.3 is more precise because its absolute error is smaller.

When we are using only the default error in stating a set of measurements, the most precise number in the set will be that number whose last significant digit, reading left to right, is in the decimal place furthest to the right, i.e., in the decimal place corresponding to the smallest unit, since that will be the number for which the absolute error, represented by the default error, will be smallest.

Example: Consider the two approximate numbers

456.32 and 973.654.

We can represent 456.32 as $456.32 \pm .005$ and 973.654 as $973.654 \pm .0005$. Since the absolute error for 973.654 is smallest, 973.654 is more precise than 456.32. However, the default absolute error is determined by the decimal place of the last significant digit. Since the last significant digit in 456.32 is in the hundredths place and the last significant digit in 973.654 is in the thousandths place, we can say that 973.654 is more precise than 456.32 without actually writing out the default error.

Example: 2340 is more precise than 5600 since the last significant digit in 2340 is in the tens place and the last significant digit in 5600 is in the hundreds place. We could check this by looking at the respective default errors. Note that we have to assume that all of these final zeros are not significant since we are not given any information that tells us that they are significant.

It is meaningless to say that a measurement is precise. We either say that one measurement is more precise than another measurement or set of measurements, as we did in the examples, or we can state that a number is precise to a specific level of precision. For example, 456.32 is *precise to the nearest hundredth* and 973.654 is *precise to the nearest thousandth*.

Accuracy is a description of relative error. The smaller the relative error, the more accurate the measurement. (In the sciences, the term *accuracy* is often used to describe the degree to which a measurement represents the true value of what is being measured.)

Example: Consider the measurements

$$246.3 \pm .4 \quad \text{and} \quad 143.78 \pm .06.$$

The relative error for the first measurement is

$$\frac{.4}{246.3} \approx .00162,$$

and the relative error for the second measurement is

$$\frac{.06}{143.78} \approx .000417.$$

Hence, $143.78 \pm .06$ is the more accurate measurement.

When we are representing a set of measurements or approximate numbers using the default error, the most accurate measurement in that set of measurements will be the one with the most significant digits in it. Since the default error is always ±.5 in the place of the last significant digit, the more significant digits, the smaller the ratio of the absolute error to the whole measurement.

Example: Consider the two measurements

758.4 and 3.28.

758.4 has four significant digits and 3.28 has three significant digits, so 758.4 is the more accurate measurement. We can check this by computing the relative error for each number. Using the default error, we can represent 758.4 by $758.4 \pm .05$ and 3.28 by $3.28 \pm .005$. Then the relative error for 758.4 is

$$\frac{.05}{758.4} \approx .0000659,$$

and the relative error for 3.28 is

$$\frac{.005}{3.28} \approx .00152.$$

This verifies that 758.4 is the more accurate number, since its relative error is smaller. However, we were able to determine that just by counting the number of significant digits in the numbers.

Note that in the example above 758.4 is the more accurate number, even though 3.28 is the more precise of the two numbers. Precision and accuracy need to be considered separately.

It is meaningless to say that a measurement is accurate. We either say that one measurement is more accurate than another measurement or set of measurements, as we did in the examples, or we can state that a number is accurate to a specific level of accuracy. For example, 758.4 is *accurate to four significant digits*, and 3.28 is *accurate to three significant digits*.

Operations with approximate numbers

When we combine approximate numbers using algebraic operations, we need to keep track of the *propagated error*. There are three standard ways of doing this: interval analysis (IA), approximate absolute error (AAE), and significant digit (SiD). We will use primarily the significant digit method.

(IA) *Interval analysis.* In this method, at each stage we compute what the least and greatest possible values of the quantity are. The answers are represented by intervals.

Example: (428.12±.005) + (45.6±.05)

428.12±.005 can be represented by the interval [428.115, 428.125].
45.6±.05 can be represented by the interval [45.55,45.65].
Since 428.115 + 45.55 = 473.665 is the smallest value that the sum can take on and 428.125 + 45.65 = 473.775 is the greatest value that the sum can take on, the answer to the problem can be represented by the interval [473.665, 473.775], meaning the sum of the two approximate numbers has to be in that interval.

Example: (138.5±.05) × (9.63±.005)

138.5±.05 can be represented by the interval [138.45, 138.55].
9.63±.005 can be represented by the interval [9.625, 9.635].
Since 138.45 × 9.625 = 1332.58125 is the smallest value that the product can take on and 138.55 × 9.635 = 1334.92925 is the greatest value that

the product can take on, the product can be represented by the interval [1332.56125, 1334.92925], meaning that the product of the two approximate numbers has to be in that interval.

This method gets even more complicated when we include the operations of subtraction and division and when we work with expressions with several different operations.

(AAE) *Approximate absolute error*. By keeping track of the absolute and relative errors, we can give an approximate (maximum possible) absolute error (or relative error) for our final answer. The answer is represented by the computed value ± the approximate absolute error. There are alternatives to the two rules given below, but these are perhaps the most common ways to estimate the approximate absolute (and relative) errors.

AAE Rule 1. When we add or subtract, we add the absolute errors.

Example: (428.12±.005) + (45.6±.05)
Since .005+.05 = .055, (428.12±.005) + (45.6±.05) = 473.72 ± .055.

Example: (237±.5) – (125.2±.05)
Even though we are subtracting, we add the absolute errors, so
(237±.5) – (125.2±.05) = 111.8 ± .55.

AAE Rule 2. When we multiply or divide, we add the relative errors.

Example: (138.5±.05) × (9.63±.005)
138.5 × 9.63 = 1333.755.
The relative error for 138.5±.05 is

$$\frac{.05}{138.5} \approx .000361 .$$

The relative error for 9.63 ± .005 is

$$\frac{.005}{9.63} \approx .000519 .$$

Then adding the two relative errors, we get .000361+.000519 = .000880.
To get the approximate absolute error for the product, we multiply the relative error times the computed product, so
.000880 × 1333.755 = 1.1737044.
So, (138.5±.05) × (9.63±.005) = 1333.755 ± 1.1737044.

(SiD) *Significant digits.* When we are using only the default errors in our measurements, we can get an answer whose default error approximates the error

of the computed quantity by keeping track of the number of significant digits and the decimal position of the last significant digit.

Rule 1. When approximate numbers are added or subtracted, the result is expressed with the precision of the **least** precise number.

Example: 428.12 + 45.6

428.12 + 45.6 = 473.72. 428.12 is precise to the nearest hundredth and 45.6 is precise to the nearest tenth. Since we are adding, we round the answer to the nearest tenth, the level of precision of the least precise of the two numbers. Hence, we represent the answer as 473.7

Example: 237 – 125.2

237 – 125.2 = 111.8. 237 is precise to the nearest unit, and 125.2 is precise to the nearest tenth. Since we are subtracting, we round the answer to the nearest unit, the level of precision of the least precise of the numbers. Hence, the answer is represented as 112.

Rule 2. When approximate numbers are multiplied or divided, the result is expressed with the accuracy of the **least** accurate number.

Example: 138.5×9.63

$138.5 \times 9.63 = 1333.755$. 138.5 is accurate to four significant digits, and 9.63 is accurate to three significant digits. Since we are multiplying, we round the answer to the level of accuracy of the least accurate of the numbers. Since 9.63 is the least accurate of the numbers, we round our answer to three significant digits, and hence we represent our answer as 1330 – with only three significant digits.

Example: 2.34/1.7

$\frac{2.34}{1.7} = 1.37647...$. 2.34 is accurate to three significant digits, and 1.7 is accurate to two significant digits. Since we are dividing, we round the answer to the level of accuracy of the least accurate of the two numbers. Since 1.7 is the least accurate of the numbers, we round our answer to two significant digits. Therefore, we represent our answer as 1.4 – with only two significant digits.

Rule 3. When the root of an approximate number is found, the result is expressed with the accuracy of the original number.

Example: $\sqrt{57.43}$

$\sqrt{57.43} = 7.578258375\ldots$ Since 57.43 has four significant digits, the root should also have four significant digits. So we round our answer to four significant digits and represent our answer as 7.578.

Rule 4. When an exact number is used in a computation involving approximate numbers, the exact number does not affect the precision or accuracy of the answer. They are determined by the approximate numbers involved.

Example: Suppose we want to find the average of the two approximate numbers 1.78 and 1.91. Then we need to compute the quantity $\frac{1.78+1.91}{2}$. In this formula, the 2 is understood to be an exact number and does not affect the precision or accuracy of the final answer.

$$\frac{1.78+1.91}{2}=\frac{3.69}{2}=1.845$$

When we add 1.78 and 1.91, the answer will be represented as 3.69, which is precise to the nearest hundredth. 3.69 has three significant digits. When we divide by the exact number 2, we will not change the level of accuracy and hence we will retain three significant digits. Hence, the answer we give is 1.85.

EXERCISES:

1. Determine the number of significant digits in the following approximate numbers:
   a. 3.456
   b. .00056
   c. 625000
   d. 0.24080
   e. 759.009
   f. 53200.

2. Round the following numbers to three (3) significant digits.
   a. 45.673 b. .0034914 c. 245681

   d. 999.872 e. 129.65 f. 479.35

3. For each of the following approximate numbers (measurements), find the default error and the corresponding interval of numbers represented by the approximate numbers.
   a. 45.67
   b. .000362
   c. 2.405
   d. 350
   e. 350.
   f. 27000

4. Suppose that the exact length of a metal bar is 48 cm, but that we have recorded a measurement for the length of the bar as 51 cm. Describe this measurement error in two different ways (not just different words for the same thing). Don't worry about why there is an error; concentrate on ways to describe it.

5. Find the absolute error, relative error, and percentage error for each of the following.

   a. For the measured value 3.21, if the exact value is 3.15.
   b. For the measured value 1.06, if the exact value is 1.24.
   c. For the measured value 741, given the set of measured values
      741, 769, and 785.

6. Find the relative and percentage errors for each of the following, using the measured value to approximate the exact value.

   a. $4.72 \pm .25$
   b. $31.0 \pm .5$
   c. 2342 ± 45
   d. 45 ± 12

7. State the level of precision and the level of accuracy of each of the following approximate numbers.

   a. 3.48
   b. 0.02305
   c. 12.9820
   d. 120
   e. 120.
   f. 35,600

8. For each of the following sets of numbers, determine which number is more precise and state the reason why.

   a. 34.591 ; .000234
   b. 45,600 ; 32,000
   c. 15.7 ; 321.85
   d. 845.7 ; 2452.6
   e. .0023 ; .052

9. For each of the following sets of numbers, determine which number is more accurate and state the reason why.

   a. 34.591 ; .000234
   b. 45,600 ; 32,000
   c. 15.7 ; 321.85
   d. 845.7 ; 2452.6
   e. .0023 ; .052

10. Complete the following computations and round the answers according to the SiD rules for operations with approximate number.

    a. 147.27 + 5.806
    b. 3.8 + 0.154 + 47.26
    c. 4700 + 23000
    d. 6.82 + 14.231
    e. 468.14 – 36.7
    f. 49.54 +3.715 – 23.4623
    g. (704.65)(0.38)
    h. 246.31×3.47
    i. (3.63) (17.064) (18.88)
    j. (452) (2300)
    k. $608.47 \div 3.92$
    l. $5.689 \div 2.23641$

m. $\frac{(67.831)\ (24.3)}{46.47}$

n. $\sqrt{19.3}$

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Answers to error analysis handout problems

1. a. 4 significant digits
   b. 2 significant digits
   c. 3 significant digits
   d 5 significant digits
   e 6 significant digits
   f 5 significant digits

2. a. 45.7
   b. .00349
   c. 246000=2.46×10^5
   d. 1.00×10^3 (Note that 1000 is ambiguous and 1000**.** is not correct.)
   e. 130**.** = 1.30×10^2 (Note the decimal point in 130**.)**
   f. 479

3. (a) $\pm .005$ (b) $\pm .0000005$ (c) $\pm .0005$ (d) ± 5 (e) $\pm .5$(f) ± 500

4. Abs error = 3cm, Rel error ≈ 0.06 or 6%

5. (a) Abs error = 0.06, Rel error ≈ 0.02 or 2%
   (b) Abs error = -0.18, Rel error ≈ 0.15 or 15%
   (c) Exact value ≈ 765, Abs error ≈ -24, Rel error ≈ 0.031 or 3.1%

6. (a) 00.053 or 5.3% (b) 0.02 or 2% (c) 0.019 or 1.9% (d) 0.27 or 27%

7. (a) precise to the nearest hudredth, accurate to 3 SiD
   (b) precise to the nearest 100 thousandth, accurate to 4 SiD
   (c) precise to the nearest 10 thousandth, accurate to 6 SiD
   (d) precise to the nearest ten, accurate to 2 SiD
   (e) precise to the nearest one, accurate to 3 SiD
   (f) precise to the nearest hundred, accurate to 3 SiD

8. (a) .000234 (precise to nearest millionth)
   (b) 45,600 (precise to nearest hundred)
   (c) 321.85 (precise to nearest hundredth)
   (d) equally precise (to the nearest tenth)
   (e) .0023 (precise to nearest 10 thousanth)

9. (a) 34.591 (accurate to 5 SiD)
   (b) 45,600 (accurate to 3 SiD)
   (c) 321.85 (accurate to 5 SiD)
   (d) 2452.6 (accurate to 5 SiD)
   (e) equally accurate (to 2 SiD)

10. All answers should be preceded by the ≈ sign.

(a) 153.08
(b) 51.2
(c) 28000
(d) 21.05
(e) 431.4
(f) 29.79
(g) 270
(h) 855
(i) 1170
(j) $1,\bar{0}00,000 = 1.0 \times 10^6$
(k) 155
(l) 2.544
(m) 35.5
(n) 4.39