Isaac Newton (1642–1727)

Isaac Newton was born on Christmas day, the posthumous child of a yeoman father and Hannah (née Ayscough) Newton. At birth he was a physical weakling who, it was said, could have fit into a quart mug. He grew up in his father's house near the hamlet of Woolsthorpe in Lincolnshire. When he was three, his mother remarried and he was placed in the care of his maternal grandmother for eight years. The English Civil War had begun. The parliamentary forces under Cromwell were defeating Charles I. Raids by armed men were frequent, and even in places where there was no immediate danger, people lived in fear. Isaac, a solitary child without playmates, turned to meditation. He began to construct mechanical contrivances—fiery kites, lanterns, and windmills. When her second husband died, his mother returned to Woolsthorpe in 1653, intending to make a farmer of Isaac. His uncle and the master of Grantham School convinced her that the boy was unsuited for such work and should be sent to a university.

In 1661, Newton was admitted to Trinity College, Cambridge, as a sizar, a student who did chores for wealthier students. He received a scholarship in 1664. Newton received the B.A. degree in 1665, when a bout of the great plague was beginning in England. As the disease spread to many parts of the country, universities were closed. Some students moved with tutors to nearby villages, but from June 1665 to 1667, Newton stayed home in Woolsthorpe. During that time, he made at least one visit to Cambridge. In his elderly musings, Newton would remember this period as an *anni mirabiles* during which he laid the foundations for his revolutionary scientific achievement. In mathematics he discovered by induction the general binomial theorem and invented an early stage of calculus. In a crucial experiment in optics, he decomposed white light into a spectrum of colors with a prism and recombined it with a second. In mechanics he posited the inverse-square law of gravitational attraction. His extraordinary scientific creativity was under way. His studies of calculus and spectroscopy were in embryo, and in mechanics he continued to examine the Cartesian vortex theory, which he later rejected in favor of gravitational attraction. After returning to Trinity College in 1667, he completed the M.A. degree the next year, whereupon he was appointed a major fellow. In October 1669 the young and obscure Newton, without political connections, was appointed Lucasian Professor of Mathematics, a position he held until 1701. Likely, Isaac Barrow, the first Lucasian Professor, saw himself as a divine rather than a mathematician. Angling for a position with higher preferment and recognizing Newton as a prodigy, Barrow had him appointed to this prestigious chair. Few students attended Newton's lectures in algebra and dynamics; fewer still understood them. His teaching departed from the dominant Cartesian physics presented in school texts. He was a quirky, retiring scholar who was known to eat sparingly, skip meals or eat cold meals left standing for hours, forget to sleep, and be disordered in outward appearances, sometimes wearing unkempt clothing. But, more important, he was a compulsively precise scholar in his research and had an amazing ability to think on a problem to the exclusion of all else over long periods of time. He lived at Trinity until 1696 in near isolation, if not some alienation, while pursuing his research.

In the two decades after 1668, Newton began to win a reputation in learned circles as the scientific genius of Britain, a reputation recognized throughout Britain by the 1710s. During the early 1670s, he concentrated on optical research. After inventing a reflecting telescope to free telescopes from chromatic aberration, he was elected in 1672 a Fellow of the Royal Society, the highest scientific honor in Britain. Shortly thereafter, he published a new theory of light and colors based on his discovery of the chromatic composition of white light. His new theory founded the science of spectroscopy. A controversy arose when Robert Hooke accused Newton of stealing optical ideas from him. Criticisms from Hooke and others had an acid effect on Newton, who decided to publish no further works on optics until after Hooke's death, which occurred in 1702. In 1684 came the famous visit of the astronomer Edmond Halley, who asked him to prove that the inverse-square law of gravitational attraction agreed with Kepler's three planetary laws. Halley also asked him to write the *Principia Mathematica* (1687) and offered to pay for its printing. Newton incorporated Kepler's laws and Galileo's law of free fall into a general dynamics. Newton's theory first unified celestial and terrestrial dynamics under the law of gravitational attraction and justified Copernican astronomy. Motion replaced Aristotelian rest as the natural state.

After the publication of the *Principia,* Newton's activity was no longer confined to the cloistered academic world of Cambridge. He was elected a Whig member of Parliament in 1689. In London he met John Locke and Richard Bentley, with whom he discussed theological and biblical questions. In the autumn of 1693 he suffered from sleeplessness, digestive problems, memory loss, and paranoia that amounted to a breakdown. Older reasons given for this were the strain of composing the *Principia* and the university crisis with James II. The symptoms, though, match those of chronic mercury poisoning and probably reflect his exposing himself to dangerous chemicals in alchemical experiments. During his breakdown he made unjustified accusations against his friends John Locke and Samuel Pepys, accusing Locke of attempting to "embroil him with women." Locke remained a friend and did not respond, and Newton, who never married, soon recovered his equilibrium. He was appointed Warden of the Mint in 1696 and carried through a recoinage during a fiscal crisis. Old hammered coins whose edges were often clipped were replaced by coins with milled edges. Newton was reelected M.P. in 1701 but was defeated in a third election try in 1706. In 1703 he was elected president of the Royal Society, which he ruled over tightly. In 1705 Queen Anne knighted him.

In his final years, Newton was occupied with a number of scientific, mystical, and humanistic concerns. His second major book, *Opticks,* presents a corpuscular theory of light, the nutshell theory of matter, and in an appendix entitled *Tractatus de quadratura curvarum* ("Treatise on the Quadrature of Curves") on the method of fluxions. From 1711 to 1712, he guided the deliberations of the committee of the Royal Society investigating whether he or Leibniz deserved priority and originality in the invention of calculus. Newton, not the committee, wrote the *Commercium epistolicum* (1712) according Newton priority. In his last years, Newton continued studies of alchemy and hermetic philosophy that he had pursued since the 1670s, as well as studies of biblical chronology and history. He died in 1727, following a lengthy struggle with gout and inflamed lungs, and was buried in Westminster Abbey, an honor reserved previously for royalty.

Newton contributed to many branches of mathematics, including algebra and number theory, Euclidean and analytic geometry, finite differences, methods of approximation, and probability. In algebra, for example, he almost completely classified cubics, obtaining seventy-two species, but lacked detailed proofs, which James Stirling provided. Among Newton's writings on mathematics are *Arithmetica Universalis* (1707), *Methodus Differentialis* (1711), and *The Method of Fluxions and Infinite Series* (publ. posth., 1736). According to Newton, his chief influence on mathematics comes from his theory of infinite series and method of fluxions, an early stage of calculus. Building on Wallisian methods of interpolation and extrapolation; the study of expansions of integrals $(1 - t^2)^n$ for $n = 1, 2, 3$, and 4, which showed the initial terms of the series increasing in odd powers and having alternate signs; and brilliant guesses, he generalized the binomial theorem, the expansion of $(a + b)^n$. Previously, it had been known for exponents n that are positive integers; Newton in 1665 first produced binomial expansions for negative and fractional n as well. But he did not supply a rigorous proof. This was later offered by Euler for all real n and by Abel for all n of complex value. Even though Newton's binomial expansions were obtained without rigorous proofs, they were correct and put emphasis on infinite series, whose study was to dominate calculus.

It took time and intense creativity for Newton to discover and develop his fluxional calculus. In 1664, his last undergraduate year at Cambridge, he copiously studied Cartesian coordinate geometry from van Schooten's Latin edition of *La géométrie* and works of Mercator and Oughtred. He also prepared extensive notes on the infinitesimal analysis found in Wallis's *Arithmetica infinitorum* (1656) and tangent constructions in Barrow's *Lectiones* (1664). His research into the interlocking structures of these two seemingly separate branches of mathematics laid the foundations for his discovery in 1665 that integration and differentiation are directly inverse operations in his method of fluxions. His fluxion is a derivative, mainly taken with respect to time. Expressed in his dot notation $\dot{x} = dx/dt$.

Newton did not publish on his new fluxional calculus for some time. His first efforts to publish on it were thwarted by the unsalability of technical treatises and the severe depression in the book trade following the Great Fire of London (1666). Neither his fluxional tract of October 1666 nor his "De Analysis per Aequationes Infinitas" ("On Analysis by Infinite Equations," ca. 1669) appeared in print before 1700. The hail of criticism that greeted his novel corpuscular theory of light in the early 1670s and his continuing grappling with what concepts are central to the method of fluxions made him hesitant to publish on fluxional calculus. The *Principia,* written in Euclidean geometric format, was Newton's first publication to involve the method of fluxions, although it made no explicit reference to fluxions. Book One speaks of prime and ultimate ratios, essentially conceiving of the derivative of an algebraic function as

$$\lim_{h \to 0} \left\{ \frac{f(x+h) - f(x)}{h} \right\}.$$

Book Two refers to an infinitesimal or evanescent increase in x as the moment of x and denotes it by $\dot{x}o$, but Newton had already grown uneasy with infinitesimals, or differentials, that can vanish. His first full exposition of fluxional calculus, the *Tractatus de quadratura curvarum,* written about 1691, hardly mentions them. It abandons static aggregates of little zero infinitesimals as a basic for calculus and instead relies on variable quantities known as *fluents* that are generated by the continuous motion of points, lines, and planes; their rates of change, the fluxion; and prime and ultimate ratios. This exposition was not published until 1704.

Newton did not, of course, complete the development of calculus. While he was aware of the importance of convergence of infinite series, others had to develop convergency tests. Likewise, his concept of limit was hazy. Satisfactory foundations for calculus were not established until over a century later, chiefly by Augustin Cauchy and Karl Weierstrass. In providing the beginning stage of calculus, Newton went beyond classical and Hellenistic Greek geometry with an independent science whose powerful methods can handle a vastly expanded range of problems in physics.

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From a Letter to Henry Oldenburg on a General Method for Finding Quadratures[1] (October 24, 1676)*

– ISAAC NEWTON

I can hardly tell with what pleasure I have read the letters of those very distinguished men Leibniz and Tschirnhaus. Leibniz's method for obtaining convergent series is certainly very elegant, and it would have sufficiently revealed the genius of its author, even if he had written nothing else. But what he has scattered elsewhere throughout his letter is most worthy of his reputation—it leads us also to hope for very great things from him. The variety of ways by which the same goal is approached has given me the greater pleasure, because three methods of arriving at series of that kind had already become known to me, so that I could scarcely expect a new one to be communicated to us. One of mine I have described before; I now add another, namely, that by which I first chanced on these series—for I chanced on them before I knew the divisions and extractions of roots which I now use. And an explanation of this will serve to lay bare, what Leibniz desires from me, the basis of the theorem set forth near the beginning of the former letter.

At the beginning of my mathematical studies, when I had met with the works of our celebrated Wallis, on considering the series by the intercalation of which he himself exhibits the area of the circle and the hyperbola, the fact that, in the series of curves whose common base or axis is x and the ordinates

$$(1-x^2)^{0/2},\ (1-x^2)^{1/2},\ (1-x^2)^{2/2},$$
$$(1-x^2)^{3/2},\ (1-x^2)^{4/2},\ (1-x^2)^{5/2},\ \text{etc.},$$

if the areas of every other of them, namely

$$x,\ x-\tfrac{1}{3}x^3,\ x-\tfrac{2}{3}x^3+\tfrac{1}{5}x^5,$$
$$x-\tfrac{3}{3}x^3+\tfrac{3}{5}x^5-\tfrac{1}{7}x^7,\ \text{etc.}$$

could be interpolated, we should have the areas of the intermediate ones, of which the first $(1-x^2)^{1/2}$ is the circle: in order to interpolate these series I noted that in all of them the first term was x and that the second terms $\tfrac{0}{3}x^3$, $\tfrac{1}{3}x^3$, $\tfrac{2}{3}x^3$, $\tfrac{3}{3}x^3$, etc., were in arithmetical progression, and hence that the first two terms of the series to be intercalated ought to be $x-\tfrac{1}{3}(\tfrac{1}{2}x^3)$, $x-\tfrac{1}{3}(\tfrac{3}{2}x^3)$, $x-\tfrac{1}{3}(\tfrac{5}{2}x^3)$, etc. To intercalate the rest I began to reflect that the denominators 1, 3, 5, 7, etc. were in arithmetical progression, so that the numerical coefficients of the numerators only were still in need of investigation. But in the alternately given areas these were the figures of powers of the number 11, namely of these, 11^0, 11^1, 11^2, 11^3, 11^4, that is, first 1; then 1, 1; thirdly, 1, 2, 1; fourthly 1, 3, 3, 1; fifthly 1, 4, 6, 4, 1, etc. And so I began to inquire how the remaining figures in these series could be derived from the first two given figures, and I found that on putting m for the second figure, the rest would be produced by continual multiplication of the terms of this series,

$$\frac{m-0}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{4}\times\frac{m-4}{5},\ \text{etc.}$$

For example, let $m=4$, and $4\times\tfrac{1}{2}(m-1)$, that is 6 will be the third term, and $6\times\tfrac{1}{3}(m-2)$, that is 4 the fourth, and $4\times\tfrac{1}{4}(m-3)$, that is 1 the fifth, and $1\times\tfrac{1}{5}(m-4)$, that is 0 the sixth,

* SOURCE: This translation from H. W. Turnbull, F.R.S., ed., *The Correspondence of Isaac Newton*, vol. II (1960), 130–34 and 148–49. Reprinted with permission of Cambridge University Press.

at which term in this case the series stops. Accordingly, I applied this rule for interposing series among series, and since, for the circle, the second term was $\frac{1}{3}(\frac{1}{2}x^3)$, I put $m = \frac{1}{2}$, and the terms arising were

$$\frac{1}{2} \times \frac{\frac{1}{2} - 1}{2} \text{ or } -\tfrac{1}{8},$$

$$-\frac{1}{8} \times \frac{\frac{1}{2} - 2}{3} \text{ or } +\tfrac{1}{16},$$

$$\frac{1}{16} \times \frac{\frac{1}{2} - 3}{4} \text{ or } -\tfrac{5}{128},$$

and so to infinity. Whence I came to understand that the area of the circular segment which I wanted was

$$x - \frac{\frac{1}{2}x^3}{3} - \frac{\frac{1}{8}x^5}{5} - \frac{\frac{1}{16}x^7}{7} - \frac{\frac{5}{128}x^9}{3} \text{ etc.}$$

And by the same reasoning the areas of the remaining curves, which were to be inserted, were likewise obtained: as also the area of the hyperbola and of the other alternate curves in this series $(1 + x^2)^{0/2}$, $(1 + x^2)^{1/2}$, $(1 + x^2)^{2/2}$, $(1 + x^2)^{3/2}$, etc. And the same theory serves to intercalate other series, and that through intervals of two or more terms when they are absent at the same time. This was my first entry upon these studies, and it had certainly escaped my memory, had I not a few weeks ago cast my eye back on some notes.

But when I had learnt this, I immediately began to consider that the terms

$$(1 - x^2)^{0/2},\ (1 - x^2)^{2/2},$$
$$(1 - x^2)^{4/2},\ (1 - x^2)^{6/2},\ \text{etc.,}$$

that is to say,

$$1,\ 1 - x^2,\ 1 - 2x^2 + x^4,$$
$$1 - 3x^2 + 3x^4 - x^6,\ \text{etc.}$$

could be interpolated in the same way as the areas generated by them: and that nothing else was required for this purpose but to omit the denominators 1, 3, 5, 7, etc., which are in the terms expressing the areas; this means that the coefficients of the terms of the quantity to be intercalated $(1 - x^2)^{1/2}$, or $(1 - x^2)^{3/2}$, or in general $(1 - x^2)^m$, arise by the continued multiplication of the terms of this series

$$m \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4}, \text{ etc.,}$$

so that (for example)

$(1 - x^2)^{1/2}$ was the value of

$$1 - \tfrac{1}{2}x^2 - \tfrac{1}{8}x^4 - \tfrac{1}{16}x^6 \text{ etc.,}$$

$(1 - x^2)^{3/2}$ of $1 - \frac{3}{2}x^2 + \frac{3}{8}x^4 + \frac{1}{16}x^6$, etc.,

and

$(1 - x^2)^{1/3}$ of $1 - \frac{1}{3}x^2 - \frac{1}{9}x^4 - \frac{5}{81}x^6$, etc.

So then the general reduction of radicals into infinite series by that rule, which I laid down at the beginning of my earlier letter became known to me, and that before I was acquainted with the extraction of roots. But once this was known, that other could not long remain hidden from me. For in order to test these processes, I multiplied

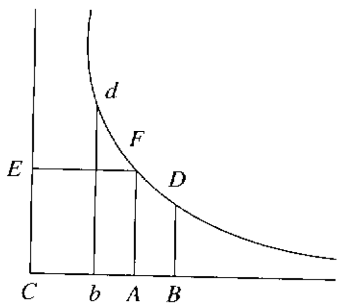
$$1 - \tfrac{1}{2}x^2 - \tfrac{1}{8}x^4 - \tfrac{1}{16}x^6, \text{ etc.}$$

into itself; and it became $1 - x^2$, the remaining terms vanishing by the continuation of the series to infinity. And even so $1 - \frac{1}{3}x^2 - \frac{1}{9}x^4 - \frac{5}{81}x^6$, etc. multiplied twice into itself also produced $1 - x^2$. And as this was not only sure proof of these conclusions so too it guided me to try whether, conversely, these series, which it thus affirmed to be roots of the quantity $1 - x^2$, might not be extracted out of it in an arithmetical manner. And the matter turned out well. This was the form of the working in square roots.

$$\begin{array}{l}
1 - x^2 (1 - \tfrac{1}{2}x^2 - \tfrac{1}{8}x^4 - \tfrac{1}{16}x^6, \text{ etc.)} \\
\underline{1\qquad\quad} \\
0 - x^2 \\
\quad \underline{- x^2 + \tfrac{1}{4}x^4} \\
\qquad -\tfrac{1}{4}x^4 \\
\qquad \underline{-\tfrac{1}{4}x^4 + \tfrac{1}{8}x^6 + \tfrac{1}{64}x^8} \\
\qquad\quad 0 \quad -\tfrac{1}{8}x^6 - \tfrac{1}{64}x^8.
\end{array}$$

After getting this clear I have quite given up the interpolation of series, and have made use of these operations only, as giving more natural foundations. Nor was there any secret about reduction by division, an easier affair in any case. But soon I attacked the resolution of affected equations and obtained it. Whence the ordinates, the segments of the axes and any other right lines at once became known from the areas or arcs of the curves being given. For the return to them needed nothing beyond the solution of the equations by which the areas or arcs were given in terms of the given right lines.

At that time the plague breaking out forced me to flee hence and think about other things. Yet, soon after, I added a certain way of finding logarithms from the area of an hyperbola, which I here append. Let dFD be an hyperbola, C its centre, F its vertex, and let $CAFE = 1$ be an inscribed square. In CA take AB, Ab, on this side and that, equal to $\frac{1}{10}$ or $0 \cdot 1$. Then, the perpendiculars BD,



bd being erected to terminate on the hyperbola, the semi-sum of the areas AD and Ad will be

$$= 0 \cdot 1 + \frac{0 \cdot 001}{3} + \frac{0 \cdot 00001}{5} + \frac{0 \cdot 0000001}{7}$$

and the semi-difference

$$= \frac{0 \cdot 01}{2} + \frac{0 \cdot 0001}{4} + \frac{0 \cdot 000001}{6} + \frac{0 \cdot 00000001}{8}, \text{ etc.}$$

which give on reduction

0 · 1000000000000	0 · 0050000000000
3333333333	250000000
20000000	1666666
142857	12500
1111	100
9	1
0 · 1003353477310	0 · 0050251679267

The sum of these, $0 \cdot 1053605156577$, is Ad, and the difference, $0 \cdot 0953101798043$, is AD. And in the same way, if AB, Ab are taken on this side and that, equal to $0 \cdot 2$, the result $Ad = 0 \cdot 2231435513142$ and $AD = 0 \cdot 1823215567939$ will be had. Thus, having obtained the hyperbolic logarithms of the four decimal numbers $0 \cdot 8$, $0 \cdot 9$, $1 \cdot 1$ and $1 \cdot 2$, since $(1 \cdot 2/0 \cdot 8) \times (1 \cdot 2/0 \cdot 9) = 2$, and $0 \cdot 8$ and $0 \cdot 9$ are less than unity, add their logarithms to twice the logarithm of $1 \cdot 2$ and you will have $0 \cdot 6931471805597$ for the hyperbolic logarithm of the number 2. To the triple of this add $\log 0 \cdot 8$ (since $(2 \times 2 \times 2)/0 \cdot 8 = 10$) and you will have $2 \cdot 3025850929933$ for the logarithm of the number 10. Thence, by addition the logarithms of the numbers 9 and 11 follow at once; so that the logarithms of all the primes 2, 3, 5, 11 are in readiness. In addition, merely by lowering the numbers in the above calculation by decimal places, and by addition, the logarithms of the decimals $0 \cdot 98$, $0 \cdot 99$, $1 \cdot 01$, $1 \cdot 02$ are obtained; as also of $0 \cdot 998$, $0 \cdot 999$, $1 \cdot 001$, $1 \cdot 002$. And then by addition and subtraction the logarithms of the primes 7, 13, 17, 37, etc., emerge. And these, combined with the above and divided by the logarithm of the number 10, become true logarithms for inserting in the table. But afterwards I have obtained them more closely.

I am ashamed to tell to how many places I carried these computations, having no other business at that time: for then I took really too much delight in these inventions. But when there appeared that ingenious work, the *Logarithmotechnia* of Nicolas Mercator (whom I suppose to have made his discoveries first), I began to pay less attention to these things, suspecting that either he knew the extraction of roots as well as division of fractions, or at least that others upon the discovery of division would find out the rest before I could reach a ripe age for writing. Yet at the very time when this book appeared, a certain compendium of the method of these series was communicated by Mr. Barrow (then professor of mathematics) to Mr. Collins; in which I had indicated the areas and lengths of all curves, and the surfaces and volumes of solids from given right lines, and that conversely from these as given the right lines could be determined; and the method there disclosed I had illustrated by various series. When afterwards a regular correspondence developed between us, Collins, a man born to promote the art of mathematics, did not cease to suggest that I should make these things public. And five years ago when, urged by my friends, I had planned to publish a treatise on the refraction of light and on colours, which I then had in readiness, I began again to think about these series and I compiled a treatise on them too, with a view to publishing both at the same time. But on the occasion of the Reflecting Telescope, when I had sent you a letter in which I briefly explained my ideas of the nature of light, something unexpected caused me to feel that it was my business to write to you in haste about the printing of that letter. Then frequent interruptions that immediately arose from the letters of various persons (full of objections and of other matters) quite deterred me from the design and caused me to accuse myself of imprudence, because, in hunting for a shadow hitherto, I had sacrificed my peace, a matter of real substance.

About that time, from just a single one of my series which Collins had sent him, Gregory, after much reflection, as he wrote back to Collins, arrived at the same method, and he left a treatise on it which we hope is going to be published by his friends. Indeed, with his strong understanding he could not fail to add many discoveries of his own, and it is in the interest of mathematics that these should not be lost. Moreover, I myself had not completely finished my treatise when I desisted from the proposal, nor has my mind to this day returned to the task of adding the rest. In fact there was wanting that part in which I had decided to explain the mode of solving problems which cannot be reduced to squarings; although I had done something to lay its foundations.

But in that treatise infinite series played no great part. Not a few other things I brought together, among them the method of drawing tangents which the very skillful Sluse communicated to you two or three years ago, about which you wrote back [to him] (on the suggestion of Collins) that the same method had been known to me also. We happened on it by different reasoning: for, as I work it, the matter needs no proof. Nobody, if he possessed my basis, could draw tangents any other way, unless he were deliberately wandering from the straight path. Indeed we do not here stick at equations in radicals involving one or each indefinite quantity, however complicated they may be; but without any reduction of such equations (which would generally render the work endless) the tangent is drawn directly. And the same is true in questions of maxima and minima, and in some others too, of which I am not now speaking. The foundation of these operations is evident enough, in fact; but because I cannot proceed with the explanation of it now, I have preferred to conceal it thus: *6accdæ 13eff7i3l9n 4o4qrr4s8t12vx*.

[*Portion of letter omitted.*]

The Anagram

This inverse problem of tangents, when the tangent between the point of contact and the axis of the figure is of given length, does not demand these methods. Yet it is that mechanical curve the determination of which depends on the area of an hyperbola. The problem is also of the same kind, when the part of the axis between the tangent and the ordinate is given in length. But I should scarcely have reckoned these cases among the sports of nature. For when in the right-angled triangle, which is formed by that part of the axis, the tangent and the ordinate, the relation of any two sides is defined by any equation, the problem can be solved apart from my general method. But when a part of the axis ending at some point given in position enters the bracket, then the question is apt to work out differently.

The communication of the solution of affected equations by the method of Leibniz will be very agreeable; so too an explanation how he comports himself when the indices of the powers are fractional, as in this equation.

$$20 + x^{3/7} - x^{6/5}y^{2/3} - y^{7/11} = 0,$$

or surds, as in

$$(x^{\sqrt{2}} + x^{\sqrt{7}})^{\sqrt[3]{2/3}} = y,$$

where $\sqrt{2}$ and $\sqrt{7}$ do not mean coefficients of

x, but indices of powers or dignities of it, and $\sqrt[3]{2/3}$ means the power of the binomial $x^{\sqrt{2}} + x^{\sqrt{7}}$. The point, I think, is clear by my method, otherwise I should have described it. But a term must at last be set to this wordy letter. The letter of the most excellent Leibniz fully deserved of course that I should give it this more extended reply. And this time I wanted to write in greater detail because I did not believe that your more engaging pursuits should often be interrupted by me with this rather austere kind of writing.

Turnbull's Note

1. Oldenburg transmitted Newton's letter of June 13, 1676 to Leibniz, who responded in a letter (August 17, 1676), revealing his results in finding quadratures and hinting that he had a general method. Leibniz's letter interested Newton, who wrote a second letter to Oldenburg dated October 24, 1676. His second letter guardedly presented by means of an anagram a general method for finding quadratures and its inverse problem of tangents.