Deconstructing Polyominoes

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Chapter 1: Introduction

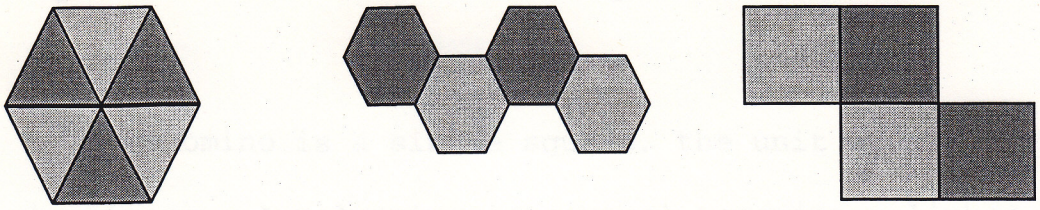
There are many systems that are composed of simple objects that communicate with one another in some way. Interesting properties of the system can be observed from the interaction of this communication. It is interesting to study the ways in which this communication can be interrupted, while paying attention to the point when communication stops. There are basically two ways in which communication can be interrupted. The first is that whatever connection a unit uses to transfer the information is prevented from doing so, the second is that the actual unit itself is unable to transfer information.

There are many examples of these kinds of systems; human intelligence itself is thought to come about through communication between neurons. Computer networks can be used to perform distributed computations that would be impossible on even the most advanced individual computer. Certain bacteria gather in colonies in a highly resilient form that is known as a biofilm in which layers of functioning cells cover other cells that are in a state of hibernation; nutrients and other substances are passed between the cells in a complicated way. Biofilms are an extremely resistant form of infection because the inert

bacteria are activated after the functioning bacteria cells on the top are killed. It would be useful to know how much one would need to break down the biofilm in order for it to no longer function.

In all these cases, some kind of information is being transferred between two similar structures. Communication between computers, and chemical activity between bacteria cells, both require that they have some kind of connection. When enough of these connections are broken, work on the computer network will not be able to occur, and the biofilm will not be able to sustain itself. "Work" in this case means communication between two or more computers or activity between bacteria cells. The minimum number of computers or bacteria cells needed for work to occur will be called the smallest useful group.

To model these kinds of systems one can use geometric structures like polyominoes, polyiamonds, or polyhexes. A polyomino is a group of equal-sized squares that are connected by common sides. Polyiamonds and polyhexes are similar to polyominoes, except polyiamonds are comprised of equal-sized equilateral triangles, and polyhexes are made of regular hexagons. Examples of each are shown in Figure 1. Each of these structures has unique qualities. For simplification, polyominoes are what I chose to study.



Polyiamond Polyhex Polyomino

Figure 1. Examples of polyshapes