Conditions for the Existence of Finite Projective Planes

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The Existence Conditions for Finite Projective Planes

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ABSTRACT

This study examines and exposits about the existence conditions of a finite projective plane, $PG_2(n)$, of a given order n. There are at least two equivalent construction methods utilizing a finite field of prime power order. Two constructions methods are illustrated for low order projective planes. Also discussed are some combinatorial objects whose existence is equivalent to the existence of $PG_2(n)$. Since projective planes are a special case of a type of combinatorial structure, a symmetric block design, some results from design theory are used to explore necessary conditions for the existence of $PG_2(n)$. Relationships between $PG_2(n)$ and certain types of error-correcting codes are discussed as well.

No further necessary conditions have been established. Several conjectures regarding the sufficiency of the known existence conditions have been found. Two non-existence results have previously been determined by exhaustion: there do not exist projective planes of orders 6 and 10. This study discusses independent methods, utilizing a computer backtrack search based on the equivalence of the existence of a set of n-1 mutually orthogonal Latin squares to that of a finite projective plane, to determine the existence of particular $PG_2(n)$.

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Introduction

Finite projective planes are both geometric and combinatorial objects. Background of real geometry is included to give a context for some of the combinatorial structures discussed later. This study explores finite projective planes as a type of combinatorial structure called a design. The concepts of combinatorial design theory that characterize finite projective planes were not studied rigorously until the 19th century. The methods section discusses in detail the design theoretic context that is the perspective for much of this study. Included in the methods section is a summary of the existence conditions of symmetric block designs and their incidence matrix requirements. Next is the relation of the incidence matrix requirements of symmetric block designs to some coding theoretic concepts. Following the section on coding theory, finite projective planes are characterized as symmetric block designs. The relation of complete sets of mutually orthogonal Latin squares and finite projective planes motivates the discussion of both the history of search results as well as an algorithm for determining the existence of a finite projective plane. Finally, brief preliminary conclusions and anticipated results are discussed.

Background

When a person sees an object, light is actually reflected from the object to the perceiver's eyes. If we think of light as rays, our eyes see these reflected rays as points. On the other hand, when a person looks at an object, we may imagine a ray to project from the eye to the object. The appearance of objects to the human eye is characterized by the latter. Obviously, one cannot see all the light that is reflected from an object, but we can see things that are in line with a ray projected from our eyes. This is called the line of sight. Along this line of sight, things that have distance between them in space may occupy the same portion of the viewing field. Euclid knew