Southern Connecticut State University

MAT 245–Differential Equations

I. **Description**

(A) **Catalog Description**: Study of ordinary differential equations. Thorough investigation of first order equations. Additionally, second order linear equations with constant coefficients, systems of differential equations, Laplace transforms, and numerical methods.

(B) **Expanded Description**: MAT 245 involves a thorough investigation of first order differential equations, both linear and nonlinear, including solution methods, graphical representation, applications to the sciences, and numerical methods. Linear second order constant coefficient equations are exhaustively studied, and their application to vibrations is covered. First order systems of two differential equations are introduced with emphasis on phase plane diagrams, critical points and stability, and solutions methods for linear systems. Laplace transforms are studied with the ultimate goal of solving linear constant coefficient equations with discontinuous forcing functions. Throughout the course, a computer algebra system will be used to enhance learning and extend the realm of differential equations whose exact solutions will be sought.

II. **Credit**

(A) MAT 245 carries three (3) semester hours of college credit.

(B) MAT 245 is required of all mathematics majors getting the B.S. in Applied Mathematics degree.

(C) MAT 245 does not satisfy the All-University requirement in mathematics.

III. **Prerequisite**

(A) The prerequisite course is MAT 151, Calculus II. Specifically, the following material is prerequisite: Differential and Integral Calculus

(B) No placement test or skills test is necessary.

IV. **Format**

(A) MAT 245 is primarily a lecture-based course.

(B) A graphing calculator is required.

(C) Use of a computer algebra system is required.

(D) MAT 245 should be scheduled for 75-minute sessions in order to allow full development of certain concepts without interruption.

V. **Outline**

(A) First-order Ordinary Differential Equations (35%)

1. Solution methods for separable and linear equations.
2. Direction fields.
3. Equilibrium and stability.
4. Substitution/Change of variable methods (at least one method should be covered).
5. At least two applications of first-order equations. Topics could be chosen from the following list:
   - Exponential growth and decay
   - Newton's second law of motion–velocity/acceleration problems
   - Newton's law of cooling
   - Mixing problems (compartmental analysis)
   - The logistic model
6. Euler's method, the improved Euler method, and the Runge-Kutta method
7. Exact equations (optional)

(B) Second-order Equations (25%)

1. Solution methods for linear, homogeneous equations with constant coefficients.

2. Solution methods for linear, nonhomogeneous equations with constant coefficients.
    - Method of undetermined coefficients.
    - Variation of parameters.
3. Applications to mechanical systems.
4. Forced oscillations and resonance.
5. Applications to electrical circuits (optional, but should at least be mentioned).

(C) Systems of differential equations (20%)

1. Phase plane diagrams for both linear and nonlinear systems.
2. Solution method for linear systems.
3. Critical points and stability for both linear and nonlinear systems.

(D) Laplace transforms. (20%)

1. Definition of the Laplace transform.
2. Properties of the Laplace transform.
3. Solutions of linear, constant coefficient second-order equations with discontinuous forcing functions.

VI. **Proposed Texts**

Edwards, C. H., and Penney, D. E., <u>Differential Equations and Boundary Value Problems, Computing and Modeling, Third Edition</u>, Prentice Hall, 2004.
Recommended sections (about 25 sections):

- Chapter 1: all of 1.1-1.5, one topic from 1.6
- Chapter 2: all of 2.2 and 2.4-2.6, selected applications from 2.1-2.3
- Chapter 3: all of 3.1-3.6, mention section 3.7
- Chapter 5: all of 5.1-5.2
- Chapter 6: all of 6.1-6.2
- Chapter 7: all of 7.1, enough of 7.2-7.5 to be able to solve a linear, constant coefficient second-order equation with a discontinuous forcing function.

Blanchard, P., Devaney, R. L., and Hall, G. R., Differential Equations, Second Edition, Brooks/Cole, 2002. (Note: The third edition will be available in 2006.)
Recommended sections (about 26 sections):

- Chapter 1: all of 1.1-1.8, one topic from 1.9
- Chapter 2: all of 2.1-2.2
- Chapter 3: all of 3.1-3.6
- Chapter 4: all of 4.1-4.3
- Chapter 5: all of 5.1
- Chapter 6: all of 6.1-6.3
- Chapter 7: all of 7.1-7.3

VII. **Bibliography**

- Differential Equations, Computing and Modeling, third ed., C. Edwards and D. Penney, Pearson Education, Inc., 2004.
- Elementary Differential Equations, W. Kohler and L. Johnson, Pearson Education, Inc. 2003.
- Differential Equations: A Modeling Approach, G. Ledder, The McGraw-Hill Companies, Inc., 2005.
- Fundamentals of Differential Equations, sixth ed., Nagle, Saff, and Snider, Pearson Education, Inc., 2004.
- Elementary Differential Equations, W. Trench, Brooks/Cole, 2000.
- Introduction to Differential Equations and Dynamical Systems, second ed., R. Williamson, The McGraw-Hill Companies, Inc., 2001.

VIII. **Outcomes**
Students passing MAT 245 should be able to do each of the following tasks.
By hand:

(A) Solve a first-order ODE using the following techniques: separation of variables, method for linear equations, a substitution/change of variable method.

(B) Solve a linear, second-order equation with constant coefficients (both homogeneous and non-homogeneous).

(C) Set up and solve first and second order equations that model an application.

(D) Compute a few iterations of Euler's approximation method for first order equations.

(E) Solve a two-by-two linear system of first order equations.

(F) Find and analyze critical points for two-by-two systems (both linear and non-linear).

(G) Compute the Laplace transform using the definition for a simple function.

(H) Using tables, find Laplace transforms and inverse Laplace transforms.

(I) Use Laplace transforms to solve a linear, constant coefficient second-order equation with a "nice" forcing function.

Using technology (Computer Algebra System):
Note: Computer algebra system tutorials and sample lab assignments are available on the K: drive in the Course Resources/MAT 245 folder.

(A) Obtain direction fields and phase plane diagrams.

(B) Solve first-order ODEs using Euler's method, the improved Euler method, and the Runge-Kutta method. Compare these three methods for speed of convergence. Students should be required to program at least two of the methods, perhaps given an example of a program for one of the other methods.

(C) Solve differential equations of all types (first-order, second-order, systems, homogeneous, non-homogeneous, nonlinear) using the differential equation solver.

(D) Use Laplace transforms to solve a second-order equation with a discontinuous forcing function.

IX. **Waiver Policy** There is no standard waiver exam for MAT 245. This course can only be waived with special consent of the department chairperson in consultation with the professor in charge of the course.

X. **Prepared and Approved**

Prepared on September 30, 2005.
Approved by the Mathematics DCC on October 4, 2005.
Approved by the Mathematics Department on November 9, 2005.

XI. **Preparers**

Therese Bennett
Len Brin
Ross Gingrich
John Scheuermann