MAT 573 Algebraic Structures I

Department of Mathematics

Southern Connecticut State University

I. Catalog Description

Linear Algebra: systems of linear equations, matrix algebra, vector spaces and linear transformations. Introduction to Group Theory: subgroups, structure of cyclic groups, direct products, group isomorphisms, cosets and Lagrange's theorem. Prerequisite: Grade of C or higher in MAT 250. Scheduled fall semesters of odd years. 4 credits.

II. Purpose

Mathematics 573 is one of several content courses designed for the Mathematics Department's Certification Program. Algebra lies at the heart of the secondary school curriculum and this course covers much of the algebra content recommended for prospective secondary school teachers.

III. Number of Credits

4 credits

IV. Prerequisites

Grade of "C" or higher in MAT 250.

V. Format

MAT 573 is primarily a lecture-based course.

VI. Course Objectives

Upon completion of MAT 573, students will be able to:

(A) Demonstrate the ability to perform basic computational tasks related to groups and vector spaces (e.g. solve systems of linear equations, find various vector spaces related to matrices, carry out calculations related to subgroups and cyclic groups, etc).

(B) Show that they have internalized the interplay between linear independence and spanning in a vector space.

(C) Demonstrate a basic but solid knowledge of mathematical objects encountered during their high school and undergraduate math career thus far (such as numbers, matrices, polynomials, functions, sets) and recognize the algebraic similarities they share.

(D) Understand the concept of binary operation on a set and the idea of an algebraic structure (a set endowed with operations that satisfy certain axioms).

(E) Prove elementary facts about vector spaces and groups by logically combining definitions and theorems.

(F) Make connections between groups and familiar geometric objects such as a regular n-gon, tetrahedron, cube and so on (via the dihedral and symmetric groups).

(G) Show that they acquired knowledge about the ways to obtain new algebraic structures out of old ones (e.g. sub-structures and direct products).

(F) Demonstrate that they learned new ways of denoting familiar objects (e.g. cyclic notation for permutations).

(H) Understand the transfer of algebraic properties from one structure to another (via linear transformations and group homomorphisms).

(I) Recognize when two algebraic structures are abstractly "the same" (in other words, internalize the concept of isomorphism).

VII. Outline

(A) Matrices and Systems of Linear Equations (30%)

1. Systems of linear equations and their matrix representations.
2. Solving systems of linear equations: Gaussian elimination.
3. Matrix operations.
4. Inverses of matrices.
5. Determinants and their properties and applications.

(B) Vector Spaces and Linear Transformations (30%)

1. Vector spaces and subspaces.
2. The span of a set of vectors.
3. Linear independence and linear dependence.
4. Basis and dimension.
5. Linear transformations.

(C) Groups (40%)

1. Definition and elementary properties of a group.
2. Main examples of groups coming from geometry (e.g. the dihedral groups D_n), number systems (including complex numbers), sets, polynomials, functions, matrices, $\mathbb{Z}_n$, etc.

3. Subgroups (including the center, centralizers, conjugate subgroups), subgroup tests, and order of elements.
4. Cyclic groups and their properties.
5. Groups of permutations S_n and A_n, and cyclic notation.
6. Group isomorphisms and their properties, automorphisms and inner automorphisms.
7. Cosets and their properties, Lagrange's Theorem and some of its consequences.

VIII. Recommended Texts

(A) D. Lay, *Linear Algebra and its Applications*, 4th Edition, Pearson.

(B) Spence, Insel, and Friedberg, *Linear Algebra with Applications*, 2nd Edition, Pearson.

(C) J. Gallian, *Contemporary Abstract Algebra*, 8th Edition, Brooks/Cole.

(D) T. Hungerford, *Abstract Algebra: An Introduction*, 3rd Edition, Brooks/Cole.

(E) J. Fraleigh, *A First Course in Abstract Algebra*, 7th Edition, Addison and Wesley.

IX. Bibliography

(A) J.A. Beachy, W. D. Blair, *Abstract Algebra*, 3rd Edition, Waveland Press.

(B) P. E. Bland, *The Basics of Abstract Algebra*, 1st Edition, W.H.Freeman.

(C) N. J. Bloch, *Abstract Algebra with Applications*, 1st Edition, Prentice-Hall.

(D) J. Durbin, Modern Algebra, an Introduction, 6th Edition, Wiley.

(E) F. M. Goodman, *Algebra, Abstract and concrete*, University of Iowa (open source).

(F) A. P. Hillman, G. L. Alexanderson, *Abstract Algebra, A First Course*, 5th Edition, Waveland Press.

(G) B. Kolman and D. Hill, *Elementary Linear Algebra with Applications*, 9th Edition, Pearson.

(H) S. J. Leon, *Linear Algebra with Applications*, 8th Edition, Pearson.

(I) W. K. Nicholson, *Linear Algebra with Applications*, 6th Edition, McGraw-Hill Ryerson.

(J) W. K. Nicholson, *Introduction to Abstract Algebra*, 4th Edition, Wiley.

(K) R. H. Redfield, *Abstract Algebra, a Concrete Introduction*, 1st Edition, Addison-Wesley.

(L) J. Rotman, *A First Course in Abstract Algebra with Applications*, 3rd Edition, Pearson.

(M) L. Rowen, *Groups, Rings and Fields*, 1st Edition, A.K. Peters (now open source).

X. Prepared and Approved

Proposed outline prepared by A. D'Amour and J. Hong, February 2014.

Approved by the GPC, March 6, 2014.

Approved by the Department of Mathematics, May 2014.