MAT 574 Algebraic Structures II

Department of Mathematics
Southern Connecticut State University

I. Catalog Description

Normal subgroups and factor groups; rings, ideals and factor rings; polynomial rings and irreducibility; extension fields; finite fields; introduction to Galois theory. Prerequisite: Grade of "C" or higher in MAT 573 or department permission. Scheduled spring semesters of even years. 3 credits.

II. Purpose

MAT 574 continues the study of groups up to and including quotient groups started in MAT 573. But for the most part, it covers some basic principles of ring and field theory that pertain to the theory of equations. This course is required for students enrolled in the Master of Science Degree in Mathematical Education Program. Definitions are carefully stated, most theorems carefully proven, and abundant concrete examples given. The student should begin to see the type of questions asked by algebraists and how algebra interacts with other branches of mathematics. Upon completion of this course, the student should understand the basic connections between some of the major concepts of modern abstract algebra and the classical polynomial algebra covered in high school, as well as the relationship of algebra to constructive geometry. Such understanding will give teachers of secondary mathematics a unifying structural overview of the subject they teach and will provide all the students with an important example of the power of abstraction in mathematics.

III. Number of Credits:

3 credits

IV. Prerequisites

Grade of "C" or higher in MAT 573 or department permission.

V. Format

MAT 574 is primarily a lecture-based course.

VI. Course Objectives

Upon completion of this course, students will be able to:

(A) Demonstrate the ability to perform basic computational tasks related to groups, rings and fields (e.g. number theoretical consequences, orders of sub-structures, degrees of extensions, etc).

(B) Show that they possess a basic but solid knowledge of mathematical objects encountered during their high school and undergraduate math career thus far (such as numbers, matrices, polynomials, functions, sets) and recognize the algebraic similarities they share.

(C) Prove elementary facts about groups, rings and fields by logically combining definitions and theorems.

(D) Show that they acquired knowledge about the ways to obtain new algebraic structures out of old ones (e.g. sub-structures, direct products, quotients).

(E) Understand the transfer of algebraic properties from one structure to another (via group and ring homomorphisms).

(F) Recognize when two algebraic structures are abstractly "the same" (in other words, internalize the concept of isomorphism).

(G) Make connections between the abstract structural properties studied in this course and some of the more concrete algebraic properties taught in high school algebra.

(H) Understand in broad terms the reasons behind the insolvability of the general quintic.

VII. Outline

(A) Groups (20%)

1. Direct products and their properties.
2. Normal subgroups, factor groups, and Cauchy's Theorem.
3. Group homomorphisms and their properties.
4. First Isomorphism Theorem, correspondence of normal subgroups and kernels, and double-quotient theorem.

(B) Rings and Fields (15%)

1. Definition and elementary properties of a ring.
2. Main examples of rings including number systems (e.g. Gaussian integers), matrices, polynomials, functions, etc.
3. Subring, direct sum of rings, and types of elements (e.g. unit, zero-divisor, idempotent, nilpotent).
4. Types of rings: commutative, integral domain, division ring, field, characteristic of a ring, finite domain is a field.

5. Field of quotients (Optional).

(C) Ideals and Quotient Rings (20%)

1. Ideals, prime ideal and maximal ideal.
2. Chain conditions: Noetherian and Artinian rings (Optional).
3. Quotient rings.
4. Ring homomorphism, isomorphism, and automorphisms.
5. Isomorphism theorems.

(D) Polynomial Rings, Factorization, Arithmetic in Integral Domain (20%)

1. Polynomial rings, basic divisibility in $F[x]$ for F a field, factorization of polynomials and irreducibility criteria (including Eisenstein).
2. Unique factorization domains and principal ideal domains.
3. Euclidean domains and norm functions in quadratic rings (Optional).

(E) Field Extensions and Galois Theory (25%)

1. Field extension (Kronecker's Theorem), and splitting field.
2. Algebraic extension, finite extension, and their properties.
3. Constructible numbers and classic Greek problems (Optional).
4. Classification of finite fields, and their subfields.
5. Introduction to Galois theory in characteristic 0: automorphism groups, fixed fields, fundamental theorem of Galois theory, solvable group, and solvability of polynomials by radicals.

VIII. Recommended Texts

(A) T. Hungerford, *Abstract Algebra: An Introduction*, 3rd Edition, Brooks/Cole.

(B) J. Gallian, *Contemporary Abstract Algebra*, 8th Edition, Brooks/Cole.

(C) J. Fraleigh, *A First Course in Abstract Algebra*, 7th Edition, Addison and Wesley.

IX. Bibliography

(A) J.A. Beachy, W. D. Blair, *Abstract Algebra*, 3rd Edition, Waveland Press.

(B) P. E. Bland, *The Basics of Abstract Algebra*, 1st Edition, W.H.Freeman.

(C) N. J. Bloch, *Abstract Algebra with Applications*, 1st Edition, Prentice-Hall.

(D) J. Durbin, Modern Algebra, an Introduction, 6th Edition, Wiley.

(E) F. M. Goodman, *Algebra, Abstract and concrete*, University of Iowa (open source).

(F) A. P. Hillman, G. L. Alexanderson, *Abstract Algebra, A First Course*, 5th Edition, Waveland Press.

(G) W. K. Nicholson, *Introduction to Abstract Algebra*, 4th Edition, Wiley.

(H) R. H. Redfield, *Abstract Algebra, a Concrete Introduction*, 1st Edition, Addison-Wesley.

(I) J. Rotman, *A First Course in Abstract Algebra with Applications*, 3rd Edition, Pearson.

(J) L. Rowen, *Groups, Rings and Fields*, 1st Edition, A.K. Peters (now open source).

X. Prepared and Approved

Proposed outline prepared by A. D'Amour and J. Hong, February 2014.

Approved by the GPC on March 6, 2014,

Approved by the Department of Mathematics, May 2014.