MAT 104 Mathematics All Around Us

Department of Mathematics
Southern Connecticut State University

I. Catalog Description

Students taking this course will gain an appreciation of the beauty of mathematics and how mathematics applies to their lives. Mathematics in nature, solving puzzles, playing games, breaking codes, photo editing, and other familiar items take center stage. Relevant elementary and advanced mathematics follows naturally, emphasizing its wide-ranging application to the real world; developing (a) analytic skills, (b) problem solving skills, (c) inductive and deductive reasoning ability, and (d) facility with mathematical algorithms and formulas; recognizing quantitative skills are important for students that enter any field, even those that make little direct use of mathematics.

II. Credit

- MAT 104 carries 3 semester-hours of university credit.
- MAT 104 satisfies the University's Liberal Education Program Quantitative Reasoning requirement.

III. Prerequisites

MAT 100 or MAT 100P or MAT 102 or appropriate mathematics placement.

IV. Format

MAT 104 is primarily a hands-on course with class time provided for activities, discussion and problem solving.

V. Liberal Education Program

This course satisfies the University's Liberal Education Program (LEP) requirement in Quantitative Reasoning (QR). It addresses the key elements of the QR requirement as indicated in the Course Objectives section. Further, as a Tier 1 LEP course, it will do the following:

(A) Address at least one *Area of Knowledge and Experience* through the choice of data sets or word problems that are applied to one of the sciences, *Natural World I: Physical Realm* or *Natural World II: Life and the Environment*. For example, exploring polyhedra will increase students' aptitude for 3D visualization as might be helpful in biology or chemistry, and understanding the equation $d = rt$ is a direct nod to physics.

(B) Incorporate at least one *Discussion of Values*. For example, *Environmental Awareness* could be discussed through modeling the heights of trees using similar triangles or discovering the volume of a ton of carbon dioxide through the exploration of proportionality. *Rational Thought*

can be emphasized by asking students to interpret and make predictions from a given mathematical model. For example, students might be asked to discuss their comfort with cyber security after a discussion of encryption algorithms in use. *Aesthetic Sensitivity* is another area that could be discussed throughout the study of checkerboard coverings, color transformations of digital images, or the inconceivable shapes of rep-tiles. *Human Diversity* could be addressed in the discussion of the gender biased nature of body mass index.

(C) Address at least one *Embedded Competency* in a significant manner. Instructors may choose to address this requirement by incorporating one of the following requirements into their course: *Oral Communication* by requiring students to present the results of their work through oral presentations, *Interpersonal Effectiveness* by requiring students to work in group settings, *Information Literacy* by requiring students to research an extension of one of the topics covered in class, or *Creative Thinking* by requiring students to create and answer their own follow-up questions to class activities.

(D) Present the Quantitative Reasoning aspects of the course in context.

VI. Quantitative Reasoning Key Elements

(A) **QR1**: Quantitative Situations - Identifying the essential quantitative elements in both routine and novel situations and understanding the relationships between those quantitative elements, and producing mathematical models appropriate for the intended analysis (e.g., writing equation(s) to represent the situation).

(B) **QR2**: Quantitative Data - Representing quantitative information in both technical and common language by using symbolic, graphical, and tabular formats, and drawing correct inferences from quantitative information through the interpretations of such representations.

(C) **QR3**: Methods - Acquiring the tools and methods necessary to resolve both routine and novel quantitative questions, including a correct sequencing of procedures, and using them appropriately, given the nature and constraints of a situation. In addition to using knowledge previously acquired in intermediate algebra, students will demonstrate proficiency with information presented in numerical or statistical form and mathematical concepts of growth and decay with their applications (e.g., linear, quadratic, exponential, etc.).

(D) **QR4**: Reliability of Data and Solutions - Correctly evaluating the level of accuracy stated or implied for given data, and assessing the correctness and accuracy of an analysis, including the assessment of the method and model used and the reasonableness of the solution.

(E) **QR5**: Mathematical Process - Using discovery (e.g., exploration and pattern-recognition), conjecture, and testing to develop mathematical formulas, theorems, and then giving persuasive mathematical arguments to establish their validity.

VII. Course Objectives

In addition to satisfying LEP Tier 1 requirements, MAT 104 has some specific course objectives. The Key Elements of Quantitative Reasoning(QR) are referenced. By the end of the course, a

successful student should be able to do the following:

1. Understand the role of mathematics as a language with which to express physical laws and scientific concepts. Students should also learn to use terminology and notation correctly as their knowledge is devalued if they cannot properly communicate it to others. In particular, notation provides us with precise and succinct identification of concepts, objects, and processes, and in its clearest forms, a path to follow. (QR 2)

2. Set up and use simple mathematical models. In particular, students should be able to translate word problems into corresponding mathematical problems and then interpret the results in terms of the conditions of the word problems. (QR 1, QR 2, QR 3, QR 4, QR 5)

3. Recognize variational relationships among quantities. Students should be able to express these relationships both in words and in formulas using symbolic notation. (QR 1, QR 2, QR 3, QR 4, QR 5)

4. Make a hypothesis through inductive reasoning and show it to be true through valid deductive reasoning. (QR5)

5. Use linear transformations, proportional reasoning, the counting principle, modular arithmetic, the area model, and the solution of linear equations in the context of applications. (QR 1, QR 3)

6. Execute Polya's problem solving method with a focus on the fourth principle: look back. (QR 4)

VIII. Outline

Problem Solving ($\sim$ 20%): Motivated by, e.g., nonogram puzzles, Minesweeper, sudoku, and solving algebraic equations, students will

(a) utilize Polya's four steps of problem solving

(b) explore the logic of compound statements

(c) identify valid and invalid arguments

Geometry of the World Around Us ($\sim$ 40%): Motivated by, e.g., bucky balls, dice, a dodecahedral calendar, broccoli, nautilus shells, rep-tiles, land surveying, the Pythagorean theorem, Heron's formula, one's age in seconds, body mass index, and distance/rate questions, students will

(a) identify geometric objects observed in everyday life

(b) use geometric understanding in problem solving

(c) use mathematical models to better understand the world

(d) use scaling and proportional reasoning in problem solving

(e) understand unit conversions

Algebra and Combinatorics in Our Lives ($\sim$ 40%): Motivated by, e.g., photo editing, cryptography, pick-a-number games, self-similar objects, and infinity, students will

(a) utilize the fundamental counting principle in enumeration

(b) explore alternative arithmetic models (e.g. modular arithmetic)

(c) experience linear transformations in visual and computational settings

(d) understand the inverse property as applied to objects other than real numbers

(e) understand one-to-one functions and their place in cardinality

IX. Sample Explorations

The Marathon Runners

One marathon runner holds a steady 9 minute per mile pace for the whole race. Another runner holds a steady 10 minute per mile pace for the first half of the marathon and a steady 8 minute per mile pace for the second half. Who wins? *Primary concepts to be developed*: using mathematical models to better understand the world and using geometric understanding in problem solving.

Pitch, the Card Game

In the card game pitch, players need to count points at the end of a hand. Each ace is worth 4 points, each king is worth 3, each queen is worth 2, each jack is worth 1, and each 10 is worth 10. One player has 2 aces, 1 king, 2 queens, a jack, and a ten. The other player has one ace, one king, one queen, one jack and two tens. Without counting either player's points, who has more? *Primary concept to be developed*: understanding one-to-one functions and their place in cardinality.

Eenie-meenie-minie-moe

In using the "eenie-meenie-minie-moe" song to choose which player will be "it", the leader sings the song while pointing to the players, one at time, clockwise starting with the player on their left. Whoever is pointed to last, exits the circle and is not it. The last player remaining in the circle is it. If there are 7 players in the circle, and the leader points 23 times during each incantation, which player will be it? *Primary concept to be developed*: exploring alternative arithmetic models.

Softballs and Baseballs

A softball is 9.7 cm in circumference and has a mass of 180 grams. A baseball is 7.4 cm in diameter and has a mass of 145 grams? The have the same type of covering, but are they made of the same material inside? *Primary concepts to be developed*: using scaling and proportional reasoning in problem solving, identifying the geometric objects observed in everyday life, and using mathematical models to better understand the world.

Enchiladas

A local Mexican restaurant makes four kinds of enchiladas (bean, cheese, beef, and chicken) and three different enchilada sauces (salsa, verde, and ranchera). Their enchiladas banderas plate comes with any three different enchiladas, and each enchilada is covered with a different sauce. How many different enchiladas banderas plates are there to choose from? *Primary concept to be developed*: utilizing the fundamental counting principle in enumeration.

The Distance between Earth and Moon

The distance between Earth and Moon is approximately 238,855 miles. How many centimeters is that? *Primary concept to be developed*: understanding unit conversions.

Multiple-character Cipher

Decode the message

-97 -40 17 -207 -185 -68 -416 -303 -97 -312 -178 -28 -143
-68 10 -190 -93 2 -305 -149 -7 102 134 83 -257 -194 -65.

It was coded using the matrix

$$\begin{bmatrix} -7 & 3 & 2 \\ -4 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

Primary concepts to be developed: experiencing linear transformations in a computational setting and understanding the inverse property as applied to objects other than real numbers.

X. Assessment

Individual instructors may vary assessment modes, but typically grades will be based on a combination of homework assignments, class activities, and exams.

	QR 1 Quantitative Situations	QR 2 Quantitative Data	QR 3 Methods	QR 4 Reliability of Data and Solutions	QR 5 Mathematical Process
Homework	Individual instructors decide which QR will be assessed appropriately.				
Class Activities	Individual instructors decide which QR will be assessed appropriately.				
Tests	✓	✓	✓	✓	✓
Final Exam	✓	✓	✓	✓	✓

(A) *The Heart of Mathematics: An Invitation to Effective Thinking*, Fourth Edition, Edward B. Burger, Michael Starbird, Wiley, 2012.

(B) *A Survey of Mathematics with Applications*, Allen R. Angel, Christine D. Abbott, Dennis Runde, Pearson, 2017.

XI. Waiver Policy

MAT 104 may be waived.

XII. Preparation

- Proposed outline prepared by Leon Brin, December 3, 2020.
- Approved by the MDCC, .
- Approved by the Mathematics Department, .