Southern Connecticut State University Mathematics 250 - Foundations of Mathematics

I. Description:

- 1. Catalog description: A bridge between calculus and upper level mathematics courses. Logic, sets, relations, functions, methods of proof.
- 2. Extended description: The course uses examples from calculus, elementary number theory, geometry, discrete mathematics, basic abstract algebra and linear algebra. Emphasis is on concepts that will be encountered in later undergraduate courses. Essential topics are:
 - (a) Logic (7.5%)
 - (b) Methods of Proof (30%)
 - (c) Set Theory (7.5%)
 - (d) Relations (15%)
 - (e) Functions (15%)
 - (f) Technical Writing (25%)

II. Credit

- A MAT 250 carries four semester-hours of credit
- B MAT 250 does not satisfy any LEP requirement
- C MAT 250 is required of all mathematics majors.
- D A student cannot earn more than 5 credits combined for MAT 250 and MAT 178.

III. Prerequisites

A C- or better in MAT 151

- IV. **Purpose** Mathematics 250 provides an introduction to certain fundamental concepts of mathematics. The concepts are ones that every mathematics major is expected to know and are used in nearly all upper-level undergraduate and graduate mathematics courses. Mathematics 250 is an intermediate-level course lying between the practically-oriented calculus sequence and the theoretically-oriented upper-level courses. Alongside the foundational content, Mathematics 250 also emphasizes the precision of communicating mathematics, both written and oral.
- V. Format MAT 250 is a three-contact-hour lecture format course together with a fourth hour of in-class proof writing and cooperative learning activities.
- VI. Course Outcomes Students in MAT 250 should achieve several departmental and program objectives. The objectives below are cross-referenced with the NCTM Standard 13. Students will be able to:
 - 1. State and apply logic and set operation, properties of number systems, and properties of functions and relations. (CAEP A.1.2, A.6.3)
 - 2. Explore ideas within mathematical structures presented to form, investigate and prove conjectures. (CAEP Standard 2a)
 - 3. Demonstrate the ability to write mathematical proofs. (CAEP Standard 2b)
 - 4. Communicate effectively and explain mathematics both verbally and in writing. (CAEP Standard 2d)

VII. Outline

- 1. Logic
 - (a) Propositions and Connectives
 - (b) Conditionals and Biconditionals
 - (c) Quantifiers
- 2. Methods of Proof
 - (a) Proving universal statements

- i. Trivial Proofs
- ii. Vacuous Proofs
- iii. Direct Proofs
- iv. Proofs by Contraposition
- v. Proofs by Contradiction
- vi. Proofs by Cases
- (b) Disproving universal statements
- (c) Proving existential statements
 - i. Constructive Existential Proofs
 - ii. Non-constructive Existential Proofs
- (d) Disproving existential statements
- (e) Proving and disproving statements with both quantifiers
- (f) Proofs using the principle of mathematical induction

3. Set Theory

- (a) Basic notations and operations
- (b) Set identities
- (c) Indexed families of sets
- (d) Power sets

4. Relations

- (a) Properties of relations (symmetry, anti-symmetry, transitivity, reflexivity, etc.)
- (b) Equivalence relations
- (c) Ordering relations

5. Functions

- (a) Terminology (domain, codomain, range, image, preimage, etc.)
- (b) Restrictions, extensions, projections, compositions
- (c) Injectivity, surjectivity, bijectivity

VIII. Recommended Texts

- 1. A Transition to Advanced Mathematics, by D. Smith, M. Eggen, R. St. Andre, 8th Edition, Cengage, 2015.
- IX. Waiver Policy This course cannot be waived
- X. **Preparation** Revised by Braxton Carrigan 2018.

XI. References

- 1. A Gentle Introduction to the Art of Mathematics, by J. Fields.
- 2. The Structure of Proof, by M. O'Leary, Prenctice Hall, 2002.
- 3. Transition to Higher Mathematics (Structure and Proof), by B. Dumas, J. McCarthy, McGraw-Hill, 2007.
- 4. Journey into Mathematics, An Introduction to Proofs, by J. Rotman, Prentice-Hall, 1998.