Southern Connecticut State University School of Arts and Science Department of Mathematics

Mathematics 329 – Bayesian Analysis and Decision Making

I. Description.

Catalog Description:.

Departmental Description: The course is an introduction to Bayesian analysis and inference. It covers many of the topics covered in a standard frequentist survey course (such as MAT 221) from a Bayesian perspective. Topics include gathering datasets, summarizing datasets, probability, Bayes Theorem, Bayesian inference for discrete random variables, Bayesian inference for continuous random variables, comparing Bayesian and frequentist approaches to inference, Bayesian prediction intervals for a single mean and single proportion, Bayesian inference for two means and two proportions, and Bayesian inference for linear regression.

II. Credit.

Mathematics 329 carries 3 semester-hours of University credit.

III. Prerequisites.

Mathematics 329 has a prerequisite of MAT 221 (Intermediate applied Statistics) or equivalent.

IV. Purpose.

To introduce students to the basics of the Bayesian methodology. We will consider probability and statistical inference, making the course valuable to students interested in applied statistics, data science, engineering, and computer science.

V. Format.

Mathematics 329 may follow a lecture format with homework assignments. Use of a computer package is required. A computer lab session is recommended. The course may also be offered as a hybrid course, where some sessions meet virtually.

VI. Outline. (Assuming 42 hours less 6 hours for exams/review in addition to the Final exam for a total of 36 hours)

A. Design of Experiments and Descriptive Statistics: (8%)

- 1. Sampling from a population
- 2. Types of studies
- 3. Measures of central tendency
- 4. Measures of spread
- 5. Graphical displays of datasets

B. Probability: (12%)

- 1. Probability Axioms
- 2. Conditional Probability and Independence
- 3. Bayes Theorem
- 4. Joint Probabilities

C. Discrete Random Variables: (12%)

- 1. Generic Probability Distributions
- 2. Binomial Random Variables
- 3. Hyper-Geometric Random Variables
- 4. Joint Random Variables

D. Bayesian Inference for Discrete Random Variables: (16%)

- 1. The Prior and the Posterior
- 2. Bayesian inference for generic discrete random variables
- 3. Bayesian inference for binomial random variables
- 4. Bayesian inference for hyper-geometric binomial random variables
- 5. Bayesian consequences

E. Continuous Random Variables: (12%)

- 1. Density functions
- 2. The uniform, beta and normal distributions
- 3. Joint Continuous Random Variables
- 4. Joint mixed distributions

F. Bayesian Inference for a Binomial Proportion: (12%)

- 1. Beta and Uniform priors
- 2. Prior selection
- 3. Summarizing the Posterior
- 4. Estimation and credible intervals

G. Comparing Bayesian and Frequentist Inferences (8%)

- 1. Interpretations
- 2. Estimation
- 3. Hypothesis Testing

H. Bayesian Inference for a Mean: (12%)

- 1. Discrete Prior
- 2. Continuous Prior
- 3. Choosing the prior
- 4. Predictive Density for the next observation

5. Comparing Bayesian and Frequentist Inferences for a mean

I. Bayesian Inference for the Difference between Means: (8%)

- 1. Independent Random Samples
- 2. Paired Difference Experiments
- 3. Difference between two proportions

J. Bayesian Inference for Linear Regression: (8%)

- 1. Least Squares Regression
- 2. Exponential Growth Model
- 3. Assumptions
- 4. Bayes Theorem for the Regression Model
- 5. Predicting Future Observations

VII. Objectives.

Students in MAT 329 should achieve several objectives. These objectives, provided by the Mathematics Department are listed below. Also listed are the topics from Section VI which satisfy the objectives.

Mathematics Department Objectives

- 1. Demonstrate that they can model and solve problems that represent a wide variety of realistic applications. (Applied Math Goal 1, Topics D and E)
- Appreciate the beauty, joy, and challenge in mathematics and experience mathematics as an engaging field with contemporary open questions. (Department Goal 8, Topics D, E and F)
- 3. Think analytically and critically and be able to formulate problems, solve them, and interpret their solutions. (<u>Department Goal 9, Topics D, E and F</u>)

Students successfully completing MAT 329 will be able:

To understand Bayes Theorem and how to apply it to probability problems. To solve problems using jointly distributed discrete random variables. To solve problems using jointly distributed continuous random variables. To apply and interpret Bayesian Inference for a Binomial proportion. To apply and interpret Bayesian Inference for a population mean. To apply and interpret Bayesian Inference for the difference of population means.

To apply and interpret Bayesian Inference for linear regression models. To understand the differences between Bayesian and Frequentist approaches to inference.

VIII. Sample Texts.

Introduction to Bayesian Statistics, 3rd Edition, by William M. Bolstad, James M. Curran. Wiley 2016.

IX. Waiver Policy.

Mathematics 329 cannot be waived.

X. Bibliography.

Carlin, Bradley P. and Thomas A. Louis. Bayes and Empirical Bayes Methods for Data Analysis, Chapman and Hall 1996.

Leonard, Thomas and John S. J. Hsu. Bayesian Methods. Cambridge University Press 1999.

Congdon, Peter. Applied Bayesian Modelling. Wiley 2003.

XI. Prepared.

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XII. Preparer.

Ray Mugno